KEMENTERIAN PENDIDIKAN MALAYSIA

QS 015

Mid-Semester Examination

Semester I

Session 2015/2016

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1. Simplify $\frac{3+\sqrt{3}}{2+\sqrt{3}} - \frac{1-\sqrt{3}}{3-\sqrt{3}}$ in the form $a + b\sqrt{c}$ where $a, b\ anc\ c\ \in \mathcal{R}$.

2. Obtain the solution set for $x - 1 \leq x^2 + 3x \leq x + 3$.

3. (a) Write $z = -\sqrt{2} - \sqrt{2}\, i$ in the polar form.

   (b) Express $\frac{z\,\bar{z} - 5i}{2+i}$ in the form $a + bi$ where $z = -1 + 3i$ and $\bar{z}$ is a conjugate of z.

4. Solve $log_3\,(3x + 10) - 1 = \frac{3}{log_2 3} - log_3 3x$.

5. (a) Given an arithmetic series is $\left(\frac{1}{12}\right) + \left(-\frac{1}{6}\right) + \left(-\frac{5}{12}\right) + \left(-\frac{2}{3}\right) + \ldots + \left(-\frac{43}{6}\right)$.

   Find

   (i) The number of terms in the above series.

   (ii) The sum of all terms.

   (b) (i) Express $(16 + 32x)^{\frac{3}{4}}$ in the form $a(1 + bx)^{\frac{3}{4}}$ where $a\ an\ b \in \mathcal{R}$. Hence, find the expansion of $(16 + 32x)^{\frac{3}{4}}$ in ascending powers of x up to the term in x^3.

   (ii) By substituting x = 0.01, evaluate $(1.02)^{\frac{3}{4}}$ correct to three decimal places.

6. (a) Given matrix $A = \begin{bmatrix} 10 & 7 & 4 \\ 10 & 5 & 2 \\ 5 & 4 & 3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} -7 & 5 & 6 \\ 20 & -10 & -20 \\ -15 & 5 & 20 \end{bmatrix}$ such that AB=mI, where m is a constant and I is the 3 x 3 identity matrix. Determine the value of m and deduce A^{-1}.

(b) A factory produces three new paints, P, Q and R by mixing white, red and yellow colours of paint according to a certain amount. The amount of colours(in litre) for a tin of paint is given in the following table:

	White(litre)	Red(litre)	Yellow(litre)
P	10	7	4
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The cost to produce a tin of paint P, Q and R are RM264, RM200 and RM158 respectively while the cost of a litre of white, red and yellow paint cost RM x, Rm y and RM z respectively.

(i) Obtain a system of linear equations to represent the above information. Hence, write down the matrix equation.

(ii) By using the result from (a), determine the cost of one litre of white, red and yellow paint used in the production of the new paints.

1) Simplify $\frac{3+\sqrt{3}}{2+\sqrt{3}} - \frac{1-\sqrt{3}}{3-\sqrt{3}}$ in the form $a + b\sqrt{c}$ where $a, b\ anc\ c\ \in \mathcal{R}$.

SOLUTION

$$\frac{3+\sqrt{3}}{2+\sqrt{3}} - \frac{1-\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})(3-\sqrt{3})-(1-\sqrt{3})(2+\sqrt{3})}{6-2\sqrt{3}+3\sqrt{3}-3}$$

$$= \frac{(9-3\sqrt{3}+3\sqrt{3}-3)-(2+\sqrt{3}-2\sqrt{3}-3)}{3+\sqrt{3}}$$

$$= \frac{9-3\sqrt{3}+3\sqrt{3}-3-2-\sqrt{3}+2\sqrt{3}+3}{3+\sqrt{3}}$$

$$= \frac{7+\sqrt{3}}{3+\sqrt{3}}$$

$$= \frac{(7+\sqrt{3})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$

$$= \frac{21-7\sqrt{3}+3\sqrt{3}-3}{9-3\sqrt{3}+3\sqrt{3}-3}$$

$$= \frac{18-4\sqrt{3}}{6}$$

$$= \frac{18}{6} - \frac{4\sqrt{3}}{6}$$

$$= 3 - \frac{2\sqrt{3}}{3}$$

2) Obtain the solution set for $x - 1 \leq x^2 + 3x \leq x + 3$.

SOLUTION

$$x - 1 \leq x^2 + 3x \leq x + 3$$

$x^2 + 3x \geq x - 1$

$x^2 + 3x - x + 1 \geq 0$

$x^2 + 2x + 1 \geq 0$

$(x+1)^2 \geq 0$

$x \in \mathbb{R}$

and

$x^2 + 3x \leq x + 3$

$x^2 + 3x - x - 3 \leq 0$

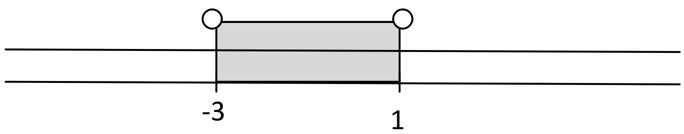
$x^2 + 2x - 3 \leq 0$

$(x+3)(x-1) \leq 0$

$x = -3, x = 1$

	$(-\infty, -3)$	$(-3,1)$	$(1, \infty)$
$(x+3)$	-	+	+
$(x-1)$	-	-	+
$(x+3)(x-1)$	+	-	+

$(-3,1)$



Therefore solution set is $\{x: -3 \leq x \leq 1\}$

3) (a) Write $z = -\sqrt{2} - \sqrt{2}\,i$ in the polar form.

(b) Express $\frac{z\,\bar{z} - 5i}{2+i}$ in the form $a + bi$ where $z = -1 + 3i$ and $\bar{z}$ is a conjugate of z.

SOLUTION

(a) $z = -\sqrt{2} - \sqrt{2}\,i$

$$r = |z| = \sqrt{\left(-\sqrt{2}\right)^2 + \left(-\sqrt{2}\right)^2} = 2$$

$$\alpha = tan^{-1}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\pi}{4}$$

$$\theta = -\pi + \frac{\pi}{4} = -\frac{3\pi}{4}$$

Polar form: $z = r(\cos\theta + i\sin\theta)$

$$= 2\left[\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right]$$

1st Q: $\theta = \alpha$

2nd Q: $\theta = \pi - \alpha$

3rd Q: $\theta = -\pi + \alpha$

4th Q: $\theta = -\alpha$

(b)
$$\frac{z\,\bar{z} - 5i}{2+i} = \frac{(-1+3i)(-1-3i) - 5i}{2+i}$$

$$= \frac{(1+3i-3i-9i^2) - 5i}{2+i}$$

$$= \frac{[1-9(-1)^2] - 5i}{2+i}$$

$$= \frac{10 - 5i}{2+i}$$

$$= \frac{(10-5i)(2-i)}{(2+i)(2-i)}$$

$$= \frac{20 - 10i - 10i + 5i^2}{4 - 2i + 2i - i^2}$$

$$= \frac{20 - 20i - 5}{4+1}$$

$$= \frac{15 - 20i}{5}$$

$$= \frac{15}{5} - \frac{20i}{5}$$

$$= 3 - 4i$$

4) Solve $log_3\,(3x+10) - 1 = \frac{3}{log_2 3} - log_3 3x.$

SOLUTION

$$log_3\,(3x+10) - 1 = \frac{3}{log_2 3} - log_3 3x$$

$$log_3\,(3x+10) - 1 = \frac{3}{\frac{log_3 3}{log_3 2}} - log_3 3x$$

$\log_a b = \frac{\log_c b}{\log_c a}$ ➔ $log_2 3 = \frac{log_3 3}{log_3 2}$

$$log_3\,(3x+10) - 1 = \frac{3log_3 2}{log_3 3} - log_3 3x$$

$\log_a a = 1$ ➔ $log_3 3 = 1$

$$log_3\,(3x+10) - 1 = \frac{3log_3 2}{1} - log_3 3x$$

$$log_3\,(3x+10) - 1 = 3log_3 2 - log_3 3x$$

$$log_3\,(3x+10) - 3log_3 2 + log_3 3x = 1$$

$a\log b = \log b^a$ ➔ $3log_3 2 = log_3 2^3$

$$log_3\,(3x+10) - log_3 2^3 + log_3 3x = 1$$

$$log_3 \frac{(3x+10)(3x)}{2^3} = 1$$

$$log_3 \left(\frac{9x^2+30x}{8}\right) = 1$$

$$\left(\frac{9x^2+30x}{8}\right) = 3^1$$

$\log_a b = c$ ➔ $b = a^c$

$$9x^2 + 30x = 24$$

$$9x^2 + 30x - 24 = 0$$

$$(9x-6)(x+4) = 0$$

$$x = \frac{6}{9} \quad or \quad x = -4$$

$$x = \frac{2}{3} \quad or \quad x = -4$$

Since $x > 0$, $\quad \therefore x = \frac{2}{3}$

5) (a) Given an arithmetic series is $\left(\frac{1}{12}\right)+\left(-\frac{1}{6}\right)+\left(-\frac{5}{12}\right)+\left(-\frac{2}{3}\right)+\ \dots\ +\left(-\frac{43}{6}\right)$.

Find

(i) The number of terms in the above series.

(ii) The sum of all terms.

(b) (i) Express $(16+32x)^{\frac{3}{4}}$ in the form $a(1+bx)^{\frac{3}{4}}$ where $a\ an\ b \in \mathcal{R}$. Hence, find the expansion of $(16+32x)^{\frac{3}{4}}$ in ascending powers of x up to the term in x^3.

(ii) By substituting $x = 0.01$, evaluate $(1.02)^{\frac{3}{4}}$ correct to three decimal places.

SOLUTION

a) i) $\left(\frac{1}{12}\right)+\left(-\frac{1}{6}\right)+\left(-\frac{5}{12}\right)+\left(-\frac{2}{3}\right)+\ \dots\ +\left(-\frac{43}{6}\right)$

$a=\left(\frac{1}{12}\right)$, $\qquad d=T_2-T_1=\left(-\frac{1}{6}\right)-\left(\frac{1}{12}\right)=-\frac{1}{4}$

$T_n=a+(n-1)d$

$$\left(-\frac{43}{6}\right)=\left(\frac{1}{12}\right)+(n-1)\left(-\frac{1}{4}\right)$$

$$\left(-\frac{43}{6}\right)=\left(\frac{1}{12}\right)+\left(\frac{1}{4}n+\frac{1}{4}\right)$$

$$-\frac{43}{6}=\frac{1}{12}+\frac{1}{4}n+\frac{1}{4}$$

$$\frac{1}{4}n=\frac{1}{12}+\frac{1}{4}+\frac{43}{6}$$

$$\frac{1}{4}n=\frac{15}{2}$$

$$n=\frac{15}{2}x\ 4$$

$$n=30$$

a) ii) $S_n=\frac{n}{2}[2a+(n-1)d]$

$$S_{30}=\frac{30}{2}\left[2\left(\frac{1}{12}\right)+(30-1)\left(-\frac{1}{4}\right)\right]$$

$$S_{30} = 15\left[\left(\frac{1}{6}\right) + (29)\left(-\frac{1}{4}\right)\right]$$

$S_{30} = \frac{-425}{4}$ or -106.25

b) i) $(16+32x)^{\frac{3}{4}} = \left[16\left(1+\frac{32x}{16}\right)\right]^{\frac{3}{4}}$

$$= 16^{\frac{3}{4}}\left(1+\frac{32x}{16}\right)^{\frac{3}{4}}$$

$$= 8(1+2x)^{\frac{3}{4}}$$

$$(16+32x)^{\frac{3}{4}} = 8(1+2x)^{\frac{3}{4}}$$

$$= 8\left[1+\frac{\left(\frac{3}{4}\right)}{1!}(2x)^1 + \frac{\left(\frac{3}{4}\right)\left(-\frac{1}{4}\right)}{2!}(2x)^2 + \frac{\left(\frac{3}{4}\right)\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)}{3!}(2x)^3\right]$$

$$= 8\left[1+\frac{3}{2}x-\frac{3}{8}x^2+\frac{5}{16}x^3+\cdots\right]$$

b) ii) $x = 0.01$

$$8(1+2x)^{\frac{3}{4}} = 8\left[1+\frac{3}{2}x-\frac{3}{8}x^2+\frac{5}{16}x^3+\cdots\right]$$

$$8[1+2(0.01)]^{\frac{3}{4}} = 8\left[1+\frac{3}{2}(0.01)-\frac{3}{8}(0.01)^2+\frac{5}{16}(0.01)^3+\cdots\right]$$

$$[1+2(0.01)]^{\frac{3}{4}} = \left[1+\frac{3}{2}(0.01)-\frac{3}{8}(0.01)^2+\frac{5}{16}(0.01)^3+\cdots\right]$$

$$[1.02]^{\frac{3}{4}} = 1.014963$$

$[1.02]^{\frac{3}{4}} = 1.015$ (3 decimal places)

6) (a) Given matrix $A = \begin{bmatrix} 10 & 7 & 4 \\ 10 & 5 & 2 \\ 5 & 4 & 3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} -7 & 5 & 6 \\ 20 & -10 & -20 \\ -15 & 5 & 20 \end{bmatrix}$ such that AB=mI, where m is a constant and I is the 3 x 3 identity matrix. Determine the value of m and deduce A^{-1}.

(b) A factory produces three new paints, P, Q and R by mixing white, red and yellow colours of paint according to a certain amount. The amount of colours(in litre) for a tin of paint is given in the following table:

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(i) Obtain a system of linear equations to represent the above information. Hence, write down the matrix equation.

(ii) By using the result from (a), determine the cost of one litre of white, red and yellow paint used in the production of the new paints.

SOLUTION

6a) $A = \begin{bmatrix} 10 & 7 & 4 \\ 10 & 5 & 2 \\ 5 & 4 & 3 \end{bmatrix}$ $\qquad B = \begin{bmatrix} -7 & 5 & 6 \\ 20 & -10 & -20 \\ -15 & 5 & 20 \end{bmatrix}$

$$AB = \begin{bmatrix} 10 & 7 & 4 \\ 10 & 5 & 2 \\ 5 & 4 & 3 \end{bmatrix}\begin{bmatrix} -7 & 5 & 6 \\ 20 & -10 & -20 \\ -15 & 5 & 20 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$AB = 10I$$

$$m = 10$$

$AB = 10I$

$A^{-1} = \frac{1}{10}B$

$A^{-1} = \frac{1}{10}\begin{bmatrix} -7 & 5 & 6 \\ 20 & -10 & -20 \\ -15 & 5 & 20 \end{bmatrix}$

$= \begin{bmatrix} -\frac{7}{10} & \frac{1}{2} & \frac{3}{5} \\ 2 & -1 & -2 \\ -\frac{3}{2} & \frac{1}{2} & 2 \end{bmatrix}$

6bi) $10x + 7y + 4z = 264$

$10x + 5y + 2z = 200$

$5x + 4y + 3z = 158$

$\begin{bmatrix} 10 & 7 & 4 \\ 10 & 5 & 2 \\ 5 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 264 \\ 200 \\ 158 \end{bmatrix}$

6bii) $X = A^{-1}D$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{7}{10} & \frac{1}{2} & \frac{3}{5} \\ 2 & -1 & -2 \\ -\frac{3}{2} & \frac{1}{2} & 2 \end{bmatrix}\begin{bmatrix} 264 \\ 200 \\ 158 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 20 \end{bmatrix}$

$\therefore x = RM10, \quad y = RM12, \quad z = RM20$