

QS 015

Mid-Semester Examination

Semester I

Session 2012/2013

1. Given that $(x + yi)(1 + i) = 1 + 2i$, find the values of x and y .
2. Solve $3 \log_8 x - \log_x 64 + 1 = 0$.
3. Solve the equation $4^{2x+1} - 65(4^x) + 16 = 0$.
4. Solve the following inequalities.
 - (a) $|2x - 3| > 5$
 - (b) $\frac{1}{x+1} \leq \frac{3}{x-1}$
5.
 - (a) A geometric sequence has seven terms. The first and the fourth terms are 8 and 216 respectively. Find the common ratio and the last term of the sequence.
 - (b) Show that the $(r + 1)^{th}$ term of binomial expansion $\left(x^2 + \frac{1}{x}\right)^{10}$ can be written as $T_{r+1} = \binom{10}{r} x^{20-3r}$. Hence, find the coefficient of x^2 .
6. Given the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.
 - (a) Find
 - (i) $|B|$
 - (ii) adjoint B
 - (iii) B^{-1}
 - (b) Hence, solve the following system of linear equations.
$$X + 2y = 2$$
$$Y + 2z = 3$$
$$X + 2y + z = 1$$

1. Given that $(x + yi)(1 + i) = 1 + 2i$, find the values of x and y .

Solution

$$(x + yi)(1 + i) = 1 + 2i$$

$$x + xi + yi - y = 1 + 2i$$

$$x - y + (x + y)i = 1 + 2i$$

$$x - y = 1$$

$$x + y = 2$$

$$\therefore x = \frac{3}{2}, y = \frac{1}{2}$$

2. Solve $3 \log_8 x - \log_x 64 + 1 = 0$.

Solution

$$3 \log_8 x - \log_x 64 + 1 = 0$$

$$3 \log_8 x - \frac{\log_8 64}{\log_8 x} + 1 = 0$$

Let $u = \log_8 x$

$$3u - \frac{2}{u} + 1 = 0$$

$$3u^2 - 2 + u = 0$$

$$3u^2 + u - 2 = 0$$

$$(3u - 2)(u + 1) = 0$$

$$u = \frac{2}{3} \quad \text{or} \quad u = -1$$

$$\log_8 x = \frac{2}{3} \quad \text{or} \quad \log_8 x = -1$$

$$x = 8^{\frac{2}{3}} = 4 \quad \text{or} \quad x = 8^{-1} = \frac{1}{8}$$

3. Solve the equation $4^{2x+1} - 65(4^x) + 16 = 0$.

Solution

$$4^{2x+1} - 65(4^x) + 16 = 0$$

$$4(4^x)^2 - 65(4^x) + 16 = 0$$

Let $u = 4^x$

$$4u^2 - 65u + 16 = 0$$

$$(4u - 1)(u - 16) = 0$$

$$4u = 1 \quad \text{or} \quad u = 16$$

$$u = \frac{1}{4} \quad \text{or} \quad 4^x = 4^2$$

$$4^x = 4^{-1} \quad \text{or} \quad x = 2$$

$$x = -1$$

$$\therefore x = -1 \quad \text{or} \quad x = 2$$

4. Solve the following inequalities.

(a) $|2x - 3| > 5$

(b) $\frac{1}{x+1} \leq \frac{3}{x-1}$

Solution

(a) $|2x - 3| > 5$

$2x - 3 > 5$ or $2x - 3 < -5$

$2x > 8$ or $2x < -2$

$x > 4$ or $x < -1$

$\therefore \{x: x < -1 \cup x > 4\}$

(b) $\frac{1}{x+1} \leq \frac{3}{x-1}$

$\frac{1}{x+1} - \frac{3}{x-1} \leq 0$

$\frac{(x-1) - 3(x+1)}{(x+1)(x-1)} \leq 0$

$\frac{-2x-4}{(x+1)(x-1)} \leq 0$

$\frac{-2(x+2)}{(x+1)(x-1)} \leq 0$

$\frac{2(x+2)}{(x+1)(x-1)} \geq 0$

$x = -2$

$x = -1$

$x = 1$

	$(-\infty, -2)$	$(-2, -1)$	$(-1, 1)$	$(1, \infty)$
$(x + 2)$	-	+	+	+
$(x + 1)$	-	-	+	+
$(x - 1)$	-	-	-	+
$\frac{(x + 2)}{(x + 1)(x - 1)}$	-	⊕	-	⊕

$\therefore [-2, -1) \cup (1, \infty)$

5. (a) A geometric sequence has seven terms. The first and the fourth terms are 8 and 216 respectively. Find the common ratio and the last term of the sequence.
- (b) Show that the $(r + 1)^{th}$ term of binomial expansion $(x^2 + \frac{1}{x})^{10}$ can be written as $T_{r+1} = \binom{10}{r} x^{20-3r}$. Hence, find the coefficient of x^2 .

Solution

(a) $a = 8, T_4 = 216$

$$8r^3 = 216$$

$$r^3 = 27$$

$$r = 3$$

$$T_7 = 8(3)^6$$

$$= 5832$$

(b)
$$T_{r+1} = \binom{10}{r} (x^2)^{10-r} \left(\frac{1}{x}\right)^r$$
$$= \binom{10}{r} (x^{20-2r})(1)^r (x)^{-r}$$
$$= \binom{10}{r} (x^{20-3r})$$

$$x^{20-3r} = x^2$$

$$20 - 3r = 2$$

$$r = 6$$

The coefficient, $x^2 = \binom{10}{6} = 210$

6. Given the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

(a) Find

(i) $|B|$

(ii) adjoint B

(iii) B^{-1}

(b) Hence, solve the following system of linear equations.

$$X + 2y = 2$$

$$Y + 2z = 3$$

$$X + 2y + z = 1$$

Solution

(a) $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

i. $|B| = 1 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$
 $= (1 - 4) - 2(0 - 2) + 0$
 $= 1$

ii. $Adj(B) = \begin{bmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & - \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} & + \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} \\ - \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ + \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} & - \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} & + \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \end{bmatrix}^T$
 $= \begin{bmatrix} -3 & 2 & -1 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}^T$
 $= \begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$

$$\begin{aligned} \text{iii. } B^{-1} &= \frac{1}{|B|} \text{Adj}(B) \\ &= \frac{1}{1} \begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\text{(b) } \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$BX = C$$

$$B^{-1}BX = B^{-1}C$$

$$X = B^{-1}C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & -2 & 4 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 5 \\ -1 \end{bmatrix}$$

$$\therefore x = -8, \quad y = 5, \quad z = -1$$