

QS 015

Mid-Semester Examination

Semester I

Session 2011/2012

1. Given $Z_1 = 1 + 2i$ and $Z_2 = 3 - 2i$. Express $Z_1 + \frac{1}{\bar{Z}_2}$ in the form of $a + bi$ where \bar{Z}_2 is the conjugate of Z_2 .
2. Solve $4^x - 3(2^x) - 4 = 0$.
3. (a) Find the determinant of $A = \begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$.
(b) Given $a = \log_x 2$ and $b = \log_x 3$, express $\log_x \left(\frac{9\sqrt{x}}{4}\right)$ in terms of a and b .
4. Solve the inequality $\left|\frac{3-2x}{x-1}\right| > 3$.
5. (a) The sum of first n terms of an arithmetic progression is $S_n = 2n^2 + 7n$. Find the first term and the common difference. Hence, find the 8th term of the progression.
(b) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x until the term including x^3 . By substituting $x = \frac{1}{50}$, evaluate $\sqrt{26}$ correct to four decimal places.
6. Given $A = \begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$.
(a) Show that $AB = 6I$.
(b) Deduce A^{-1} .
(c) Hence, solve the following system of linear equations.
$$\begin{aligned} -4x + 2y + 2z &= 1 \\ 6x - 4y + 2z &= 2 \\ -2x + 4y - 2z &= 14 \end{aligned}$$

1. Given $Z_1 = 1 + 2i$ and $Z_2 = 3 - 2i$. Express $Z_1 + \frac{1}{\bar{Z}_2}$ in the form of $a + bi$ where \bar{Z}_2 is the conjugate of Z_2 .

Solution

$$Z_2 = 3 - 2i$$

$$Z_1 + \frac{1}{\bar{Z}_2}$$

$$= 1 + 2i + \frac{1}{3 + 2i}$$

$$= 1 + 2i + \frac{1}{3 + 2i} \left(\frac{3 - 2i}{3 - 2i} \right)$$

$$= 1 + 2i + \frac{3 - 2i}{13}$$

$$= \frac{13 + 26i + 3 - 2i}{13}$$

$$= \frac{16}{13} + \frac{24}{13}i$$

2. Solve $4^x - 3(2^x) - 4 = 0$.

Solution

$$4^x - 3(2^x) - 4 = 0$$

$$2^{2x} - 3(2^x) - 4 = 0$$

Let $y = 2^x$

$$y^2 - 3y - 4 = 0$$

$$(y - 4)(y + 1) = 0$$

$$y = 4 \quad \text{or} \quad y = -1$$

$$2^x = 4 \qquad 2^x = -1(\text{Undefined})$$

$$x = 2$$

3. (a) Find the determinant of $A = \begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$.
- (b) Given $a = \log_x 2$ and $b = \log_x 3$, express $\log_x \left(\frac{9\sqrt{x}}{4}\right)$ in terms of a and b.

Solution

$$\begin{aligned} \text{(a)} \quad |A| &= (2) \begin{vmatrix} 2 & -3 \\ -6 & 2 \end{vmatrix} - (3) \begin{vmatrix} 6 & -3 \\ 3 & 2 \end{vmatrix} + (6) \begin{vmatrix} 6 & 2 \\ 3 & -6 \end{vmatrix} \\ &= 2(4 - 18) - 3(12 + 9) + 6(-36 - 6) \\ &= -343 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \log_x \left(\frac{9\sqrt{x}}{4}\right) &= \log_x 9 + \log_x \sqrt{x} - \log_x 4 \\ &= \log_x 3^2 + \log_x x^{\frac{1}{2}} - \log_x 2^2 \\ &= 2\log_x 3 + \frac{1}{2}\log_x x - 2\log_x 2 \\ &= 2b + \frac{1}{2} - 2a \end{aligned}$$

4. Solve the inequality $\left| \frac{3-2x}{x-1} \right| > 3$

Solution

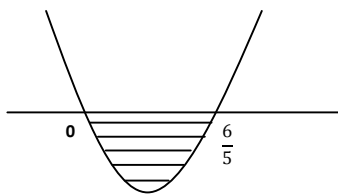
$$\left| \frac{3-2x}{x-1} \right| > 3$$

$$(3-2x)^2 > 9(x-1)^2$$

$$9-12x+4x^2 > 9(x^2-2x+1)$$

$$5x^2-6x < 0$$

$$x(5x-6) < 0$$



Therefore the solution is $(0, 1) \cup \left(1, \frac{6}{5}\right)$

5. (a) The sum of first n terms of an arithmetic progression is $S_n = 2n^2 + 7n$. Find the first term and the common difference. Hence, find the 8th term of the progression.
- (b) Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x until the term including x^3 . By substituting $x = \frac{1}{50}$, evaluate $\sqrt{26}$ correct to four decimal places.

Solution

$$\begin{aligned} \text{(a) } a &= S_1 = 2(1)^2 + 7(1) \\ a &= S_1 = 9 \\ S_2 &= 2(2)^2 + 7(2) = 22 \\ T_2 &= S_2 - S_1 = 22 - 9 = 13 \\ d &= T_2 - a = 13 - 9 = 4 \\ T_8 &= 9 + 7(4) \\ T_8 &= 37 \end{aligned}$$

$$\begin{aligned} \text{(b) } (1 + 2x)^{\frac{1}{2}} &= 1 + \frac{1}{2}(2x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2} - 1\right)}{2!}(2x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3!}(2x)^3 + \dots \\ &= 1 + x + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(4x^2) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}(8x^3) + \dots \\ &= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots \end{aligned}$$

$$\text{When } x = \frac{1}{50}$$

$$\sqrt{1 + 2\left(\frac{1}{50}\right)} = 1 + \left(\frac{1}{50}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{50}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{1}{50}\right)^3 + \dots$$

$$\sqrt{\frac{26}{25}} = 1.019804$$

$$\frac{\sqrt{26}}{5} = 1.019804$$

$$\sqrt{26} = 5(1.019804)$$

$$\sqrt{26} = 5.0990$$

6. Given $A = \begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$.

- (a) Show that $AB = 6I$.
- (b) Deduce A^{-1} .
- (c) Hence, solve the following system of linear equations.

$$-4x + 2y + 2z = 1$$

$$6x - 4y + 2z = 2$$

$$-2x + 4y - 2z = 14$$

Solution

(a) $AB = \begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 6I$$

(b) $A^{-1} = \frac{1}{6} B$

$$= \frac{1}{6} \begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{5}{6} \\ \frac{2}{3} & \frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

$$\begin{aligned}
 \text{(c)} \quad & -4x + 2y + 2z = 1 && \dots\dots\dots (1) \\
 & 6x - 4y + 2z = 2 && \dots\dots\dots (2) \\
 & -2x + 4y - 2z = 14 && \dots\dots\dots (3)
 \end{aligned}$$

$$\begin{bmatrix} -4 & 2 & 2 \\ 6 & -4 & 2 \\ -2 & 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix}$$

$$2 \begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 0 & 3 & 3 \\ 2 & 3 & 5 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 1 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ \frac{13}{2} \\ 2 \end{bmatrix}$$

$$\therefore x = 4, \quad y = \frac{13}{2}, \quad z = 2$$