

QS025/1
Mathematics
Paper 1
Semester II
Session 2014/2015
2 hours

QS025/1
Matematik
Kertas 1
Semester II
Sesi 2014/2015
2 jam



KEMENTERIAN
PENDIDIKAN
MALAYSIA

BAHAGIAN MATRIKULASI
MATRICULATION DIVISION

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **19** halaman bercetak.

This question paper consists of 19 printed pages.

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ARAHAN KEPADA CALON:

Kertas soalan ini mengandungi **10** soalan.

Jawab **semua** soalan.

Semua jawapan hendaklah ditulis pada buku jawapan yang disediakan. Gunakan muka surat baru bagi nombor soalan yang berbeza.

Markah penuh yang diperuntukkan bagi setiap soalan atau bahagian soalan ditunjukkan dalam kurungan pada penghujung soalan atau bahagian soalan.

Semua langkah kerja hendaklah ditunjukkan dengan jelas.

Kalkulator saintifik yang tidak boleh diprogramkan sahaja yang boleh digunakan.

Jawapan berangka boleh diberi dalam bentuk π , e , surd, pecahan atau sehingga tiga angka bererti, di mana-mana yang sesuai, kecuali jika dinyatakan dalam soalan.

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

SENARAI RUMUS MATEMATIK

Trigonometri

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

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$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

SENARAI RUMUS MATEMATIK

Pembezaan dan Pengamiran

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u dv = uv - \int v du$$

Sfera	$V = \frac{4}{3} \pi r^3$	$S = 4 \pi r^2$
Kon membulat tegak	$V = \frac{1}{3} \pi r^2 h$	$S = \pi r^2 + \pi r h$
Silinder membulat tegak	$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u dv = uv - \int v du$$

Sphere	$V = \frac{4}{3} \pi r^3$	$S = 4 \pi r^2$
Right circular cone	$V = \frac{1}{3} \pi r^2 h$	$S = \pi r^2 + \pi r h$
Right circular cylinder	$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$

SENARAI RUMUS MATEMATIK

Kaedah Berangka**Kaedah Newton-Raphson:**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n=1,2,3,\dots$$

Keratan Kon**Bulatan:**

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p, k) \text{ atau } F(h, k+p)$$

Elips:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h \pm c, k) \text{ atau } F(h, k \pm c)$$

LIST OF MATHEMATICAL FORMULAE

Numerical Methods

Newton-Raphson Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n=1,2,3,\dots$$

Conics

Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

$$r = \sqrt{f^2 + g^2 - c}$$

Parabola:

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

$$F(h+p, k) \text{ or } F(h, k+p)$$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$F(h \pm c, k) \text{ or } F(h, k \pm c)$$

- 1 Gunakan petua trapezium untuk menganggarkan $\int_0^1 f(x) dx$ dari data yang diberikan di bawah:

x	0.00	0.25	0.5	0.75	1.00
$f(x)$	2.4	2.6	2.9	3.2	3.6

[5 markah]

- 2 Diberi suatu parabola yang mempunyai bucu di $(-2, 1)$, membuka ke kanan dan melalui titik $(3, 6)$. Cari persamaan parabola tersebut dan tentukan fokusnya.

[6 markah]

- 3 Nilaikan kamiran yang berikut:

(a) $\int \sin 6x \cos 4x dx.$

[3 markah]

(b) $\int (3 \tan x + 4)^5 \sec^2 x dx.$

[4 markah]

- 4 Gunakan kaedah Newton-Raphson dengan anggaran awal $x_1 = 1$ untuk mencari $\sqrt[3]{2}$ dalam $[0, 2]$ betul kepada tiga tempat perpuluhan.

[7 markah]

- 1 Use the trapezoidal rule to estimate $\int_0^1 f(x) dx$ from the data given below:

x	0.00	0.25	0.5	0.75	1.00
$f(x)$	2.4	2.6	2.9	3.2	3.6

[5 marks]

- 2 Given a parabola with vertex $(-2, 1)$, opening to the right and passes through the point $(3, 6)$. Find the equation of the parabola and determine its focus.

[6 marks]

- 3 Evaluate the following integrals:

(a) $\int \sin 6x \cos 4x dx.$

[3 marks]

(b) $\int (3 \tan x + 4)^5 \sec^2 x dx.$

[4 marks]

- 4 Use the Newton-Raphson method with initial approximation $x_1 = 1$ to find $\sqrt[6]{2}$ on $[0, 2]$ correct to three decimal places.

[7 marks]

- 5 Cari persamaan sebuah bulatan $x^2 + y^2 + 2gx + 2fy + c = 0$ yang melalui titik $A(0, 1)$, $B(3, -2)$ dan $C(-1, -4)$. Seterusnya, tentukan pusat dan jejaringnya. Cari titik persilangan bulatan ini dengan paksi- y .

[10 markah]

- 6 Diberi fungsi $f(x) = \frac{1}{x+3}$ dan $g(x) = \frac{x}{4}$.

- (a) Pada paksi yang sama, lakar graf f dan g bagi nilai x di antara $x = 0$ dan $x = 2$. Lorek rantau R yang dibatasi oleh f , g , $x = 0$ dan $x = 2$.

[2 markah]

- (b) Cari luas rantau R .

[6 markah]

- (c) Cari isipadu pepejal yang terjana apabila rantau R diputar melalui 2π radian mengelilingi paksi- x .

[4 markah]

- 5 Find the equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which passes through the points $A(0, 1)$, $B(3, -2)$ and $C(-1, -4)$. Hence, determine its center and radius. Find the points of intersection of the circle with the y -axis.

[10 marks]

- 6 Given that $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{x}{4}$.

- (a) On the same axes, sketch the graphs of f and g for the values of x between $x = 0$ and $x = 2$. Shade the region R bounded by f , g , $x = 0$ and $x = 2$.

[2 marks]

- (b) Find the area of region R .

[6 marks]

- (c) Find the volume of the solid generated when the region R is rotated through 2π radian about the x -axis.

[4 marks]

- 7 (a) Amaun suatu bahan radioaktif, $Q(t)$ yang hadir pada masa t dalam satu tindak balas diberi oleh persamaan pembezaan

$$\frac{dQ}{dt} = -kQ,$$

dengan k adalah pemalar positif. Jika jisim asal bahan ini adalah 100 mg telah menyusut kepada 97 mg dalam masa 6 hari, tentukan

- (i) separuh hayat bahan tersebut.

[6 markah]

- (ii) amaun bahan radioaktif yang ada selepas 30 hari.

[2 markah]

- (b) Cari penyelesaian am untuk persamaan pembezaan

$$(1+x)\frac{dy}{dx} - y = 1+x.$$

[4 markah]

- 8 Diberi dua garis lurus,

$$L_1 : t = \frac{x-1}{-3} = \frac{y+2}{8} = \frac{z}{-3} \text{ dan } L_2 : t = \frac{x+2}{10} = \frac{y}{10} = \frac{z-4}{-7}.$$

- (a) Tunjukkan bahawa L_1 dan L_2 adalah tidak selari dan cari sudut tirus di antara dua garis lurus tersebut.

[5 markah]

- (b) Tentukan titik persilangan antara L_1 dengan satah

$$\Pi : 2x - y + 5z + 25 = 0.$$

[4 markah]

- (c) Cari satu persamaan satah yang mengandungi L_1 dan L_2 .

[4 markah]

- 7 (a) The amount $Q(t)$ of radioactive substance present at time t in a reaction is given by the differential equation

$$\frac{dQ}{dt} = -kQ,$$

where k is a positive constant. If the initial amount of the substance is 100 mg and is decreased to 97 mg in 6 days, determine

- (i) the half-life of the substance.

[6 marks]

- (ii) the amount of radioactive substance present after 30 days.

[2 marks]

- (b) Find the general solution to the differential equation

$$(1+x)\frac{dy}{dx} - y = 1+x.$$

[4 marks]

- 8 Given two straight lines,

$$L_1 : t = \frac{x-1}{-3} = \frac{y+2}{8} = \frac{z}{-3} \quad \text{and} \quad L_2 : t = \frac{x+2}{10} = \frac{y}{10} = \frac{z-4}{-7}.$$

- (a) Show that L_1 and L_2 are not parallel and find the acute angle between the two straight lines.

[5 marks]

- (b) Determine intersection point between L_1 and plane

$$\Pi : 2x - y + 5z + 25 = 0.$$

[4 marks]

- (c) Find an equation of the plane containing L_1 and L_2 .

[4 marks]

- 9 (a) Cari nilai A , B , C dan D jika

$$\frac{x^2+9}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-3)}.$$

[6 markah]

- (b) Seterusnya, nilaikan $\int_{-2}^1 \frac{x^2+9}{x^2(x-3)} dx$.

[7 markah]

- 9 (a) Find the values of A , B , C and D if

$$\frac{x^2 + 9}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-3)}.$$

[6 marks]

- (b) Hence, evaluate $\int_{-2}^1 \frac{x^2 + 9}{x^2(x-3)} dx$.

[7 marks]

10 Diberi P , Q dan R adalah tiga titik dalam suatu ruang dengan

$$\overrightarrow{PQ} = \underline{a} = 3\underline{i} - \underline{j} + \underline{k}, \quad \overrightarrow{PR} = \underline{b} = 2\underline{i} + \underline{j} - 3\underline{k}$$

dan koordinat R adalah $(3, 0, 1)$.

(a) Seterusnya, tunjukkan bahawa

(i) \underline{a} dan \underline{b} tidak seranjang.

[3 markah]

(ii) $|\underline{a} \times \underline{b}|^2 = |\underline{a}|^2 |\underline{b}|^2 - (\underline{a} \cdot \underline{b})^2$.

[5 markah]

(b) Cari luas segitiga PQR .

[2 markah]

(c) Cari persamaan Cartesian bagi

(i) satah yang melalui titik P , Q dan R .

[3 markah]

(ii) garis yang melalui titik R dan berseranjang dengan satah dalam bahagian (i).

[2 markah]

KERTAS SOALAN TAMAT

Kang Kooi Wei

10 Given P , Q and R are three points in a space where

$$\overrightarrow{PQ} = \underline{a} = 3\underline{i} - \underline{j} + \underline{k}, \quad \overrightarrow{PR} = \underline{b} = 2\underline{i} + \underline{j} - 3\underline{k}$$

and the coordinates of R is $(3, 0, 1)$.

(a) Hence, show that

(i) \underline{a} and \underline{b} are not perpendicular.

[3 marks]

(ii) $|\underline{a} \times \underline{b}|^2 = |\underline{a}|^2 |\underline{b}|^2 - (\underline{a} \cdot \underline{b})^2$.

[5 marks]

(b) Find the area of triangle PQR .

[2 marks]

(c) Find the Cartesian equation for the

(i) plane that passes through the points P , Q and R .

[3 marks]

(ii) line that passes through the point R and perpendicular to the plane in part (i).

[2 marks]

END OF QUESTION PAPER