QS025/2
Mathematics
Paper 2
Semester II
Session 2011/2012
2 hours

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Matematik
Kertas 2
Semester II
Sesi 2011/2012
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **19** halaman bercetak.
This question paper consists of **19** *printed pages.*

Kang Kooi Wei

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

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LIST OF MATHEMATICAL FORMULAE

Statistics

For ungrouped data, the kth percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\ x_{([s])}, & \text{if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[\, s\,]$ = the least integer greater than k.

For grouped data, the kth percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100}\right)n - F_{k-1}}{f_k} \right] c.$

Variance

$$s^2 = \frac{\sum f_i x_i^2 - \frac{1}{n}\left(\sum f_i x_i\right)^2}{n-1}$$

Binomial Distribution

$$X \sim B(n, p)$$

$$P(X = x) = {}^nC_x p^x (1-p)^{n-x}, \quad x = 0,1,2,3,\ldots,n$$

Poisson Distribution

$$X \sim P(\lambda)$$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,3,\ldots$$

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1 Given $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$. Calculate the probability that both events A and B occur. Hence, verify that A and B are independent events.

[6 *marks*]

2 Six yellow balls are labeled with numbers 1, 2, 3, 4, 5 and 6, and four red balls are labeled with letters P, Q, R and S. All the ten balls are of similar size. In how many different ways can one

(a) arrange all the balls in a straight line such that balls of the same colour are next to each other?

[2 *marks*]

(b) choose and arrange equal number of yellow and red balls in a straight line such that balls of the same colour are next to each other?

[4 *marks*]

3 A team of four members will be formed by selecting randomly from a group consisting of four students and six lecturers.

Calculate the number of different ways to form a team consisting of

(a) no students at all.

[2 *marks*]

(b) equal number of students and lecturers.

[2 *marks*]

(c) more students than lecturers.

[2 *marks*]

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4 The frequency distribution of the age (in years) of 80 patients in a clinic is given in the table below.

Age	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Number of Patients	5	15	24	18	10	8

Find the mean and mode. Hence, calculate and interpret Pearson's coefficient of skewness given that the standard deviation is 6.798 years.

[7 *marks*]

5 On the average, a hospital receives 6 emergency calls in 15 minutes. It is assumed that the number of emergency calls received follows the Poisson distribution.

(a) Find the probability that

(i) not more than 15 emergency calls are received in an hour.

[3 *marks*]

(ii) the hospital will receive the first emergency call between 9.00 am and 9.05 am.

[3 *marks*]

(b) Find the number of emergency calls received, *m*, if it is known that the probability at most *m* emergency calls received in half an hour is 0.155.

[4 *marks*]

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6 The following is the stem-and-leaf diagram for a sample of heights (in cm) of a type of herbal plant. All observations are integers.

13	0 6
14	5 8 8
15	0 2 5 8
16	0 6 8 8
17	1 5
19	2
23	0

(a) Calculate the mean.

[2 *marks*]

(b) Find the values of the median, first and third quartiles.

[4 *marks*]

(c) Construct the box-and-whiskers plot and comment on the data distribution.

[6 *marks*]

7 It is found that 30% of the population of an island are overweight. Among the overweight, the probability of those who do not have any chronic illness is 0.4 and among those who are not overweight, the probability that they do not have any chronic illness is 0.65.

Draw a tree diagram to represent the given information.

[2 *marks*]

(a) Hence, if a person is randomly chosen from that population, find the probability that he

(i) does not have any chronic illness.

[2 *marks*]

(ii) is overweight knowing that he does not have any chronic illness.

[2 *marks*]

(b) If two persons are randomly chosen from the population, find the probability that

(i) both of them do not have any chronic illness.

[2 *marks*]

(ii) only one of them has chronic illness.

[2 *marks*]

(iii) at least one of them does not have any chronic illness.

[2 *marks*]

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8 The probability that a type of antibiotics can cure a certain disease is 0.95.

(a) If five patients are given the antibiotics, find the probability that

(i) exactly three patients are cured after finishing the course of antibiotics.

[3 *marks*]

(ii) at least one patient is cured after finishing the course of antibiotics.

[2 *marks*]

(b) If 500 patients are given the antibiotics, find the

(i) probability that more than 480 patients are cured.

[4 *marks*]

(ii) largest possible value n such that the probability that at least n patients recovered after finishing the course of antibiotics is 0.9.

[4 *marks*]

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9 A nurse works five days in a week. The number of days in a week she works overtime is a discrete random variable X with probability function

$$f(x)=\begin{cases}\frac{k}{3}\left|3x-1\right|, & x=0,1,2\\ \frac{k}{3}(x-2), & x=3,4\\ \frac{k}{3}(x-1), & x=5\end{cases}$$

where k is a constant. Show that $k=\frac{1}{5}$.

[3 *marks*]

(a) Find the probability she works overtime everyday in a week.

[2 *marks*]

(b) Calculate the probability that she will work overtime for at least three days in a week.

[2 *marks*]

(c) Determine the most likely number of days in week she will work overtime.

[2 *marks*]

(d) Find the expected number of day in week she will work overtime. Hence, evaluate $\mathrm{E}(3X+1)$.

[4 *marks*]

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10 The continuous random variable X has the cumulative distribution function

$$F(x)=\begin{cases}0, & x<0\\ a\sqrt{x^3}-3x^2, & 0\le x\le 1\\ 1, & x>1\end{cases}$$

where a is constant. Show that $a=4$.

[2 *marks*]

Hence,

(a) calculate the mean and variance of X.

[6 *marks*]

(b) find $P\left[X-E(X)<\frac{1}{10}\right]$.

[2 *marks*]

(c) if $Y=4X-3$, find the E(Y) and Var(Y).

[5 *marks*]

END OF QUESTIONS PAPER

Kang Kooi Wei