QS026/1
Mathematics
Paper 1
Semester II
Session 2010/2011
2 hours

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Matematik
Kertas 1
Semester II
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2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This booklet consists of **15** *printed pages.*

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions in the answer booklet provided.

Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Limit

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

$$\lim_{h \to 0} \frac{1 - \cos h}{h} = 0$$

Hyperbolic

$$\sinh (x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh (x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int u\,dv = uv - \int v\,du$$

Sphere $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Right Circular Cone $V = \frac{1}{3}\pi r^2 h$ $S = \pi r s$

Right circular cylinder $V = \pi r^2 h$ $S = 2\pi r h$

1 If $x = \sec\theta$ and $y = 2\tan\theta$, find $\frac{dy}{dx}$ in terms of θ.

[5 *marks*]

2 Find an equation of the circle that passes through the points $(1,4)$, $(2,2)$ and $(-1,3)$. Hence, find the radius of the circle.

[6 *marks*]

3 Given three vectors $\underline{a} = 2\underline{i} + \beta\underline{j} + 4\underline{k}$, $\underline{b} = \underline{j} - 3\underline{k}$ and $\underline{c} = 5\underline{i} + 6\underline{j} + 2\underline{k}$. Find the value of β such that

(a) $\underline{a}$ is perpendicular to $\underline{b}$.

[3 *marks*]

(b) $\underline{a} \times \underline{b} = \underline{c}$.

[4 *marks*]

4 Prove that $\cosh^2 x - \sinh^2 x = 1$. Hence, find the value of $\tanh x$ if $\sinh x = \frac{3}{4}$.

[7 *marks*]

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5 (a) Use the first principle of derivative to show that

$$\frac{d}{dx}(\sin x) = \cos x.$$

[5 *marks*]

(b) Given $y = \sin(x^2)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. Hence, or otherwise, show that $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = 0$.

[5 *marks*]

6 Given $f(x) = \frac{9x^2 - 36}{x^2 - 9}$.

(a) Determine the vertical and horizontal asymptotes of f.

[3 *marks*]

(b) Determine the interval of x on which f is increasing and f is decreasing.

[5 *marks*]

(c) Sketch the graph of f.

[3 *marks*]

7 Given the points $A(1, 3, 1)$, $B(4, -1, 2)$, $C(12, 0, 1)$ and $D(0, 2, 0)$.

Find

(a) a vector equation of the line AB.

[3 *marks*]

(b) an equation of the plane ABC in the Cartesian form.

[5 *marks*]

(c) the acute angle between the plane ABC and the plane ABD.

[5 *marks*]

8 A circle C passes through the origin and has its centre at the point (3,-3).

(a) Obtain the equation of the circle C.

[3 *marks*]

(b) If the line $y = x - 6$ meets the circle C at the points P and Q, determine the coordinates of P and Q.

[5 *marks*]

(c) Find the coordinates of the points on the circle C where the tangents are parallel to the line PQ.

[5 *marks*]

9 The curve $y = x^4 + ax^3 + bx^2$ has a point of inflection at $(-2, 0)$.

(a) Find the values of a and b.

[5 *marks*]

(b) Show that another point of inflection of the curve is $\left(-\frac{1}{2}, \frac{15}{16}\right)$.

[4 *marks*]

(c) Use the second derivative test to find the coordinates of the local extremum of the curve.

[4 *marks*]

10 (a) Prove that $\cos 3x = 4\cos^3 x - 3\cos x$.

[4 *marks*]

(b) Use the above identity to

(i) find all the solutions in the interval $-180° < x \leq 180°$ of the equation $2\cos 3x + \cos 2x + 1 = 0$.

[7 *marks*]

(ii) show that $\cos^3 2x = \frac{1}{4}(\cos 6x + 3\cos 2x)$. Hence, evaluate $\int_0^{\frac{\pi}{6}} \cos^3 2x \, dx$.

[4 *marks*]

END OF QUESTION BOOKLET