QS026/1
Mathematics
Paper 1
Semester II
2009/2010
2 hours

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Matematik
Kertas 1
Semester II
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2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **13** halaman bercetak.
This booklet consists of **13** *printed pages.*

Kang Kooi Wei

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$
$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$
$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$
$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Limit

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$
$$\lim_{h \to 0} \frac{1 - \cos h}{h} = 0$$

Hyperbolic

$$\sinh (x + y) = \sinh x \cosh y + \cosh x \sinh y$$
$$\cosh (x + y) = \cosh x \cosh y + \sinh x \sinh y$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int u\,dv = uv - \int v\,du$$

Sphere $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Right Circular Cone $V = \frac{1}{3}\pi r^2 h$ $S = \pi r s$

Right circular cylinder $V = \pi r^2 h$ $S = 2\pi rh$

1 An ellipse with centre at the origin passes through the points $(0, 3)$ and $(1, 1)$.

Find the equation and the foci of the ellipse. [7 *marks*]

2 Prove that $1 + \tan x \tan 2x = \sec 2x$. [5 *marks*]

3 Given $\underline{u} = 2\underline{i} - 2\underline{j} + \underline{k}$. Find the vectors which have magnitude 6 and parallel to $\underline{u}$.

[6 *marks*]

4 Using the definition of hyperbolic functions, prove that

$$\frac{1}{\cosh x + \sinh x} = \cosh x - \sinh x.$$

Hence, find

$$\int \frac{1}{(\cosh x + \sinh x)^2}\, dx.$$ [7 *marks*]

5 Given that $f(x) = \dfrac{x^3 + 8}{x}$.

(a) State the asymptote of f. [1 *mark*]

(b) Find the critical and inflection points of f. [5 *marks*]

(c) Determine the intervals where f is increasing and f is decreasing.

[3 *marks*]

(d) Sketch the graph of f. [3 *marks*]

6 (a) Prove that for $\theta \neq n\pi$, where n is an integer,

$$\frac{\sin\theta}{\cos\theta\left(1-2\sin^2\theta+\sin^4\theta\right)} = \tan\theta\sec^4\theta.$$ [3 *marks*]

(b) By using (a) and the substitution $u = \tan\theta$, evaluate

$$\int_0^{\pi/4} \frac{\sin\theta}{\cos\theta\left(1-2\sin^2\theta+\sin^4\theta\right)}\, d\theta.$$ [8 *marks*]

7 The points A(1, 3, 2), B(3, −1, 6) and C(5, 2, 0) lie on the plane Π. A line L passes through the points P(1 ,2, 2) and Q(0, 1, 4). Find

(a) $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence, obtain an equation of the plane Π in Cartesian form. [7 *marks*]

(b) the parametric equations of the line L. [3 *marks*]

(a) the point of intersection of L and Π. [3 *marks*]

8 (a) If $\cos(x+y) = 2x\sin y$ where $0 \leq y \leq \pi$, find y and $\frac{dy}{dx}$ at $x = 0$. [6 *marks*]

(b) Given $y = \sin(\ln x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. [6 *marks*]

9 (a) The position of a particle moving along a straight line at any time $t > 0$ is given by $s(t) = t(t-1)(t-2)$, where s is the distance of the particle from the origin. Find the velocity of the particle at the instant when the acceleration becomes zero. [4 *marks*]

(b) A closed right circular cylindrical container of radius r and height h is to be constructed with volume 4,000 cm^3. The cost for the construction is RM 1.00 per cm^2 for the curved surface while RM 2.00 per cm^2 for the top and bottom surfaces. State h in terms of r and hence, find the radius of the cylinder so that the cost of the construction is minimum.

[8 *marks*]

10 Given the circles

$$C_1 : x^2 + y^2 - 2x - 2y + 1 = 0$$
$$C_2 : x^2 + y^2 = 1.$$

Find

(a) the centre and the radius of the circle C_1. [3 *marks*]

(b) the equations of the tangents from the point (0, 3) to the circle C_2. [5 *marks*]

(c) the equation of the circle that passes through the point (–5, 0) and the points of intersection of the circles C_1 and C_2. [7 *marks*]

END OF BOOKLET