QS026/2
Mathematics
Paper 2
Semester II
2009/2010
2 hours

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Matematik
Kertas 2
Semester II
2009/2010
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **13** halaman bercetak.
This booklet consists of **13** *printed pages.*

Kang Kooi Wei

INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left\{(y_0 + y_n) + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right)\right\}, \text{ where } h = \frac{b-a}{n}$$

Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1,2,3,\ldots$$

Statistics

For ungrouped data, the kth percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\ x_{([s])}, & \text{if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[\,s\,]$ = the least integer greater than k.

For grouped data, the kth percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100}\right)n - F_{k-1}}{f_k}\right]c$

1 Solve the differential equation $\frac{dy}{dx} = \frac{1}{\sqrt{xy}}$ with initial condition $y(0) = 4$.

Express y in terms of x. [5 *marks*]

2 Given two events A and B with

$$\mathrm{P}(B) = \frac{1}{3}, \qquad \mathrm{P}(A \cup B) = \frac{3}{4} \qquad \text{and} \qquad \mathrm{P}\left(B \middle| A\right) = \frac{1}{4}.$$

Find

(a) $\mathrm{P}(A)$. [4 *marks*]

(b) $\mathrm{P}(\overline{B} \mid \overline{A})$. [2 *marks*]

3 (a) How many one-, two-, three-, and four-digit numbers can be formed using the digits 4, 5, 6, and 7, when each digit can be used only once?

[3 *marks*]

(b) How many of the numbers formed in part (a) are odd and greater than 600?

[4 *marks*]

4 The following table represents the probability distribution of a discrete random variable Y.

Y	−2	−1	1	3	5
$\mathrm{P}(Y = y)$	0.1	0.3	0.4	0.1	0.1

Find

(a) $\mathrm{P}(|Y| > 1)$. [2 *marks*]

(b) $\mathrm{E}(Y-3)^2$ and $\mathrm{Var}(Y-3)$. [5 *marks*]

5 (a) Given $g(x) = (x-1)\sqrt{x+2}$. By using the Newton-Raphson method starting at $x_0 = 1.1$, find the root of $g(x)$. [6 *marks*]

(b) By using trapezoidal method, obtain the approximate value of $\int_0^1 x e^{x^2} dx$ based on four subintervals, correct to four decimal places.

[4 *marks*]

6 The waiting time for 50 customers to have their food served at a restaurant on a particular day is shown in the following table.

Time (minutes)	Number of Customer
1 – 5	4
6 – 10	9
11 – 15	15
16 – 20	11
21 – 25	6
26 – 30	3
31 – 35	2

(a) Calculate the mean, median and mode of the waiting time.

[8 *marks*]

(b) Plot an ogive. Hence, determine the percentage of customers who have to wait beyond 23 minutes. [5 *marks*]

7 A model for the concentration of glucose solution in the bloodstream, $C = C(t)$, is given by the differential equation $\frac{dC}{dt} = r - kC$, where r is the constant rate at which glucose solution enters the bloodstream and k is a positive constant. If $C(0) = C_0$, show that the concentration at any time t is

$$C(t) = \left(C_0 - \frac{r}{k}\right)e^{-kt} + \frac{r}{k}.$$ [8 *marks*]

After a very long period of time, the concentration of glucose is found to be 1 unit. If $C_0 = 9$, what is the concentration of glucose at $t = \frac{2}{k}$?

[4 *marks*]

8 In a class of 15 students of which 7 are males, 5 students wear spectacles. There are 3 male students who wear spectacles. Four students are chosen at random. Find the probability that

(a) all females are chosen. [2 *marks*]

(b) equal number of males and females who wear spectacles are chosen. [2 *marks*]

(c) all males are chosen if it is known that they all do not wear spectacles. [2 *marks*]

(d) more students who do not wear spectacles are chosen if it is known that they are females. [3 *marks*]

(e) all females or students who do not wear spectacles are chosen. [3 *marks*]

9 The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases}\dfrac{k\ln x}{x}, & 1<x<e\\ 0, & \text{otherwise.}\end{cases}$$

Show that $k=2$. [3 *marks*]

Hence,

(a) obtain the cumulative distribution function, $F(x)$. [3 *marks*]

(b) determine the 81^{st} percentile for the distribution of X. [3 *marks*]

(c) calculate $\mathrm{E}(X)$. [4 *marks*]

10 In any large shipment of watermelons from a particular orchard, it is known that 2% are unripe. Upon arrival of a shipment at a receiving depot, random samplings with replacement are conducted.

(a) Calculate the probability of getting at most one unripe watermelon in a sample of size 20. [4 *marks*]

(b) Approximate the probability of getting one to three unripe watermelons in a sample of size 50. [5 *marks*]

(c) If the sample size is 1000, approximate the probability of getting not more than eight unripe watermelons. [6 *marks*]

END OF BOOKLET

Kang Kooi Wei