

QS026/1
Mathematics
Paper 1
Semester II
2008/2009
2 hours

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Matematik
Kertas 1
Semester II
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2 jam



BAHAGIAN MATRIKULASI
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MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **13** halaman bercetak.
This booklet consists of 13 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Limit

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

$$\lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0$$

Hyperbolic

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

LIST OF MATHEMATICAL FORMULAE

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u dv = uv - \int v du$$

Sphere	$V = \frac{4}{3} \pi r^3$	$S = 4 \pi r^2$
Right Circular Cone	$V = \frac{1}{3} \pi r^2 h$	$S = \pi r s$
Right circular cylinder	$V = \pi r^2 h$	$S = 2 \pi r h$

1. Use the first principle of derivative to show that

$$\frac{d}{dx}(\cos x) = -\sin x.$$

[5 marks]

2. If $|\mathbf{u} + \mathbf{v}| = 5$ and $|\mathbf{u} - \mathbf{v}| = 1$, find $\mathbf{u} \cdot \mathbf{v}$ by using the property $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$.

[6 marks]

3. The end points of the diameter of a circle are A(2, 0) and B(10, 4).
Determine:

(a) the equation of the circle.

[4 marks]

(b) the equation of the tangent line to the circle at the point B.

[3 marks]

4. Show that $\frac{1}{\cosh 2x - \sinh 2x} = \cosh 2x + \sinh 2x$.

Hence, evaluate

$$\int_0^1 \frac{dx}{\cosh 2x - \sinh 2x}$$

and leave your answer in term of e .

[7 marks]

5. Water is leaking from the bottom of a conical tank with radius 1.5 meter and height 2 meter at a rate of 0.25 cubic meter per minute. The tank was initially full. If the height of water is 1 meter then find the rate of change of

(a) the water level,

[7 marks]

(b) the radius of the water surface.

[2 marks]

6. The equation $4x^2 - y^2 - 24x - 4y + 16 = 0$ represents a hyperbola.

(a) Determine the coordinates of its centre and vertices.

[7 marks]

(b) Write the equations of the asymptotes.

[2 marks]

(c) Sketch the hyperbola and label its centre, vertices and asymptotes.

[3 marks]

7. (a) By writing $\tan x = \frac{\sin x}{\cos x}$, show that $\frac{d}{dx}(\tan x) = \sec^2 x$.

Hence, find $\frac{d}{dx}(\tan(\cos 2x))$.

[6 marks]

(b) If $\sin(2xy) = x + \cos(xy^2)$, evaluate $\frac{dy}{dx}$ when $y = 0$.

[7 marks]

8. (a) Find A and B if

$$\sin 2x \cos 3x \equiv A \sin 5x + B \sin x.$$

Hence, evaluate

$$\int_0^{\pi} \sin 2x \cos 3x \, dx.$$

[7 marks]

(b) Find $\int \sin^5 2x \, dx$.

[6 marks]

9. The function f is defined by $f(x) = \frac{x^2 + x + 2}{x^2 - x + 4}$.

(a) Find the y-intercept and determine the horizontal asymptote of f .

[3 marks]

(b) Find the critical points of f and determine the intervals where f is increasing and f is decreasing.

[7 marks]

(c) Sketch the graph of f .

[3 marks]

10. Let $P(1, 3, 2)$, $Q(3, -1, 6)$, and $R(5, 2, 0)$ be points in three dimensional space.

Determine:

(a) the direction cosines for the vector \mathbf{PQ} .

[4 marks]

(b) whether \mathbf{PQ} and \mathbf{PR} are perpendicular vectors.

[4 marks]

(c) an equation of the plane containing P , Q , and R .

[4 marks]

(d) the parametric equations of the line passing through the point $B(0, 1, 2)$ and perpendicular to the plane in part (c).

[3 marks]

END OF BOOKLET