QS026/2
Mathematics
Paper 2
Semester II
2008/2009
2 hours

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Matematik
Kertas 2
Semester II
2008/2009
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

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LIST OF MATHEMATICAL FORMULAE

Trapezium Rule

$$\int_a^b f(x)\, dx = \frac{h}{2}\left\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right\}, \text{ where } h = \frac{b-a}{n}$$

Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1,2,3,\ldots$$

Statistics

For ungrouped data, the kth percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\[2ex] x_{([s])} & \text{, if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[s]$ = the least integer greater than k.

For grouped data, the kth percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100}\right)n - F_{k-1}}{f_k}\right] c$

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1. A bookstore recorded the number of books, X, sold daily with the probability
$P(X = x) = \dfrac{6-x}{20}, X = 0,1,2,3,4$. Calculate $E(X)$. Hence, find
$P\left(|X - E(X)| < \dfrac{3}{2} \right)$.

[5 marks]

2. (a) In how many ways can 6 women and 3 men be arranged in a row if

(i) the row begins with a man and ends with a woman?

[2 marks]

(ii) the men must be separated from each other?

[2 marks]

(b) In how many ways can a four-member committee be formed from 6 women and 3 men if the committee has equal number of both sexes?

[2 marks]

3. (a) Approximate $\displaystyle\int_0^1 x\sqrt{1-x^2}\, dx$ by using the Trapezoidal Rule and $n = 4$.
Give you answer correct to four decimal places.

[4 marks]

(b) Using the substitution $u = 1 - x^2$, show that $\displaystyle\int_0^1 x\sqrt{1-x^2}\, dx = \dfrac{1}{3}$.

[2 marks]

Give a reason for the difference of the values obtained in (a) and (b).

[1 mark]

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4. The selection committee of a competition will determine the winners for the first to the fifth place. If twelve females and eight males participate in the competition, in how many ways can one select

   (a) three females for the first to the third place winners, and two males for the fourth and fifth place winners?

   [2 marks]

   (b) five winners which consist of three females and two males?

   [2 marks]

   (c) at least four females win in the competition?

   [3 marks]

5. A total of 30 rats are randomly captured from a plantation and kept to breed in an experimental laboratory. After a month under observation, the number of rats has increased by 10. The rate of increase per month of the population is given by

$$\frac{dp}{dt} = kp(50 - p)$$

where p is the current population and k is a constant.

   (a) Solve the differential equation. Give your answer for p in terms of t.

   [10 marks]

   (b) Compute the number of rats after a period of one year.

   [2 marks]

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6. The following table shows the frequencies of daily income of 42 fruit sellers.

Income, x (RM)	Number of fruit sellers
$100 \leq x < 120$	4
$120 \leq x < 140$	8
$140 \leq x < 160$	14
$160 \leq x < 180$	12
$180 \leq x < 200$	4

(a) Calculate the mean and the standard deviation of the daily income.

[5 marks]

(b) What is the daily income earned by the most fruit sellers?

[2 marks]

(c) State the skewness of the daily income distribution using the Pearson's coefficient of skewness.

[3 marks]

(d) If the average income of fishmongers per day is RM180 with standard deviation of RM20, determine whose income is more stable between the fishmongers and fruit sellers?

[2 marks]

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7.   Compact discs produced by a factory are packed in boxes. Each box contains 100 compact discs. It is known that 4% of the compact discs produced are defective.

(a)   Show that the probability that a box chosen at random will contain at most 3 defective compact discs is approximately 0.43.

[3 marks]

(b)   Find the probability that among 12 boxes chosen at random, there will be 4 boxes which contain at most 3 defective compact discs.

[3 marks]

(c)   Seventy boxes are chosen at random. Find the probability that between 20 boxes and 40 boxes, inclusively, which contain at most 3 defective compact discs.

[6 marks]

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8. A total of 400 new students at a college were interviewed to find out if they either receive a scholarship, loan or no financial aid. There are 150 male students, of which 50 receive loan and 70 do not receive any financial aid. One hundred female students receive scholarship. There are 140 students who do not receive any financial aid.

If a new student is selected at random, calculate the probability that the student is a

(a) female or a scholarship recipient .

[3 marks]

(b) loan recipient if it is known that the student is a female.

[3 marks]

(c) male who is a scholarship recipient or a female who receives a loan.

[3 marks]

(d) female or non-scholarship recipient.

[3 marks]

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9. The relative frequency distribution of the marks for a Statistics test obtained by a group of 100 students in the last semester is shown in the following table.

Marks	Relative frequency
0 − 19	0.05
20 − 39	0.15
40 − 59	0.38
60 − 79	0.32
80 − 99	0.10

(a) Determine the mean and median for the distribution.

[7 marks]

(b) A student taking the statistics test is chosen at random. Using the above data, estimate the probability the student has at least 40 marks.

[2 marks]

(c) Two students taking the Statistics test are chosen at random. Using the above data, estimate the probability both have at least 40 marks.

[3 marks]

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10. A continuous random variable X has the cumulative function given by

$$F(x) = \begin{cases} 0 & ; & x \leq 1 \\ \dfrac{(x-a)^2}{12} & ; & 1 \leq x \leq 3 \\ \dfrac{14x - x^2 - 25}{b} & ; & 3 \leq x \leq 7 \\ 1 & ; & x \geq 7 \end{cases}$$

where a and b are constants.

(a) Show that $a = 1$ dan $b = 24$.

[4 marks]

(b) Find $P(2 \leq X \leq 5)$.

[2 marks]

(c) Calculate the median.

[3 marks]

(d) Determine the density function.

[3 marks]

(e) Sketch the graph of $f(x)$ and hence find the mode.

[3 marks]

END OF BOOKLET

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