

QS026/1  
Mathematics  
Paper 1  
Semester II  
2006/2007  
2 hours

QS026/1  
Matematik  
Kertas 1  
Semester II  
2006/2007  
2 jam



**BAHAGIAN MATRIKULASI**  
**KEMENTERIAN PELAJARAN MALAYSIA**  
*MATRICULATION DIVISION*  
*MINISTRY OF EDUCATION MALAYSIA*

**PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI**  
*MATRICULATION PROGRAMME EXAMINATION*

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**MATEMATIK**  
**Kertas 1**  
**2 jam**

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**JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.**  
*DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.*

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Kertas soalan ini mengandungi **13** halaman bercetak.

*This booklet consists of 13 printed pages.*

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**INSTRUCTIONS TO CANDIDATE:**

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of  $\pi$ ,  $e$ , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

## LIST OF MATHEMATICAL FORMULAE

## Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

## Limit

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

$$\lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0$$

## Hyperbolic

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

**LIST OF MATHEMATICAL FORMULAE**

**Differentiation and Integration**

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\operatorname{coth} x$	$-\operatorname{cosech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \operatorname{coth} x$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int u dv = uv - \int v du$$

<b>Sphere</b>	$V = \frac{4}{3} \pi r^3$	$S = 4 \pi r^2$
<b>Right Circular Cone</b>	$V = \frac{1}{3} \pi r^2 h$	$S = \pi r s$
<b>Right circular cylinder</b>	$V = \pi r^2 h$	$S = 2 \pi r h$

1. Given that  $\tan^{-1}(-2) = \alpha$  and  $\tan^{-1}(3) = \beta$ , find the value of  $\cot(\alpha + \beta)$ . [5 marks]

2. The vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are such that  $\mathbf{b} \times \mathbf{c} = 3\mathbf{i}$  and  $\mathbf{c} \times \mathbf{a} = 2\mathbf{j} + \mathbf{k}$ , where  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  are unit vectors. Express  $(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} + \mathbf{b} + 4\mathbf{c})$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ . [6 marks]

3. (a) Show that  $\frac{d}{dx}(\cosh x) = \sinh x$ . [2 marks]

(b) Evaluate  $\int_0^1 x \sinh x^2 dx$  to three decimal places. [5 marks]

4. Milk is being poured into a hemispherical bowl of radius 4 cm at the rate of  $3\pi \text{ cm}^3/\text{sec}$ . If the depth of the milk in the bowl is  $h$  cm, its volume  $V$  is

$$V = \pi \left( 4h^2 - \frac{h^3}{3} \right) \text{ cm}^3.$$

At the instant the milk is  $\frac{3}{2}$  cm deep, find

(a) the rate of change of  $h$ . [4 marks]

(b) the rate of change of the radius of the milk's surface. [3 marks]

5. Show that  $\sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)$ .

Hence, evaluate  $\int_0^{\frac{\pi}{6}} \cos 4x \sin 2x dx$ , giving your answer in a fraction form.

[10 marks]

6. Let  $L$  be a line passing through the centre of the circle  $x^2 + y^2 - 2x - 2y = 7$  and perpendicular to the line  $3x + 4y = 7$ . Find
- (a) the coordinates of the points of intersections of  $L$  and the circle. [9 marks]
- (b) the equations of the tangents to the circle parallel to  $3x + 4y = 7$ . [3 marks]
7. The conic section given by  $9y^2 - 4x^2 + 18y + 16x - 43 = 0$  is a hyperbola.
- (a) Express the equation in the standard form. [3 marks]
- (b) Determine the coordinates of the centre, the vertices and the foci of the hyperbola. [4 marks]
- (c) Find the equations of the asymptotes. [2 marks]
- (d) Sketch the graph. [3 marks]
8. The points  $A(-2, 1, 2)$ ,  $B(5, -7, -3)$  and  $C(3, 3, 1)$  lie on the plane  $\Pi_1$ .  
The equation of a second plane  $\Pi_2$ , is given as  $2x - y - 2z = 5$ .
- (a) Find the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ . [3 marks]
- (b) Determine the Cartesian equation of  $\Pi_1$ . [5 marks]
- (c) Find the acute angle between  $\Pi_1$  and  $\Pi_2$ , giving your answer in degrees. [5 marks]

9. The function  $f$  is defined by  $f(x) = \frac{x^2 - 4x}{x^2 + 9}$ .

- (a) Determine the asymptote(s) of  $f$ . [2 marks]
- (b) Find the critical number(s) of  $f$  and determine the intervals where  $f$  is increasing and  $f$  is decreasing. [6 marks]
- (c) Find the coordinates of the local extrema of  $f$ . [2 marks]
- (d) Sketch the graph of  $f$ . [3 marks]

10. (a) Show that  $4 \sin 2\theta + 3 \cos 2\theta - 3 \sin \theta - 3 = \sin \theta(8 \cos \theta - 6 \sin \theta - 3)$ . [3 marks]
- (b) Express  $8 \cos \theta - 6 \sin \theta$  in the form of  $R \sin(\theta - \alpha)$ , where  $R$  is positive and  $\alpha$  is an acute angle in radian. [5 marks]
- (c) By using the facts in (a) and (b), solve

$$4 \sin 2\theta = 3 \sin \theta - 3 \cos 2\theta + 3$$

for  $0 \leq \theta \leq \pi$ . Give your answers in radian correct to three significant figures.

[7 marks]

**END OF QUESTION PAPER**