QS026/2
Mathematics
Paper 2
Semester II
2006/2007
2 hours

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Matematik
Kertas 2
Semester II
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2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

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LIST OF MATHEMATICAL FORMULAE

Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left\{(y_0 + y_n) + 2\left(y_1 + y_2 + \dots + y_{n-1}\right)\right\}, \text{ where } h = \frac{b - a}{n}$$

Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1,2,3,\dots$$

Statistics

For ungrouped data, the kth percentile,

$$P_k = \begin{cases} \dfrac{x_{(s)} + x_{(s+1)}}{2}, & \text{if } s \text{ is an integer} \\[2mm] x_{([s])} & , \text{ if } s \text{ is a non-integer} \end{cases}$$

where $s = \dfrac{n \times k}{100}$ and $[\,s\,]$ = the least integer greater than k.

For grouped data, the kth percentiles, $P_k = L_k + \left[\dfrac{\left(\dfrac{k}{100}\right)n - F_{k-1}}{f_k}\right]c$

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1. The mean sample

$$w, \ w, \ 7, \ 9, \ 11, \ 12, \ 3w, \ 18$$

is 10.25 with w constant.

(a) Find w. [2 marks]

(b) Calculate the coefficient of variation and interpret your answer.

[5 marks]

2. Given two events A and B with the following probabilities:

$$P(A) = \frac{2}{5}, \quad P(\overline{A}|B) = \frac{3}{5}, \quad P(A \cap B) = \frac{1}{15}.$$

Find

(a) $P(B)$. [4 marks]

(b) $P(A|\overline{B})$. [3 marks]

3. Determine the general solution of the differential equation $\dfrac{dy}{dx} = 3xy\sqrt{x^2 - 1}$. Hence, find the equation of the curve that passes through the point $(1, -1)$. [7 marks]

4. Ali plans to buy a computer and the probability that he gets a loan is 0.6. The probability that he will buy the computer if he gets the loan is 0.9, and the probability that he will still buy the computer even though without getting the loan is 0.7.

(a) What is the probability that Ali will buy a computer? [3 marks]

(b) If it is known that Ali did not buy a computer, what is the probability that he failed to get the loan? [3 marks]

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5. **(a)** Use the trapezoidal rule with $n = 4$ to approximate $\int_0^1 \dfrac{dx}{1+x}$. Using definite integration, find the value of $\int_0^1 \dfrac{dx}{1+x}$. Compare the two answers and give a reason for the difference. [7 marks]

(b) Approximate $\sqrt[3]{7}$ by using Newton-Raphson method and initial value 2, up to the second iteration. [3 marks]

6. The following table shows the distribution for the number of medical leaves (in days) taken within a certain period by 65 employees of a company.

Number of Medical Leaves (days)	Number of Employees
1 – 3	4
4 – 6	6
7 – 9	8
10 – 12	12
13 – 15	18
16 – 18	11
19 – 21	6

(a) Calculate

(i) the mean, mode and median. Hence, describe the distribution of the data. [7 marks]

(ii) the variance. [2 marks]

(b) Without using graphs, find m such that 20% of the number of medical leaves exceeds m. [4 marks]

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7. A total of 13 candidates comprising of 5 mathematicians and 8 physicists will be selected to form a committee. In how many ways can

   (a) a committee of 5 members can be formed if it consists of at least 3 mathematicians? [4 marks]

   (b) all the candidates be placed in a row such that they always sit in a group of the same expertise? [2 marks]

   (c) any 3 members may be selected from the candidates for the positions of president, secretary and treasurer? [2 marks]

   (d) a committee of 5 members with a mathematician as president and a physicist as secretary can be formed? [2 marks]

8. Consider a simple electric circuit with the resistance of 3 Ω and inductance of 2 H. If a battery gives a constant voltage of 24 V and the switch is closed when $t = 0$, the current, $I(t)$, after t seconds is given by

$$\frac{dI}{dt} + \frac{3}{2}I = 12, \quad I(0) = 0.$$

   (a) Obtain $I(t)$. [7 marks]

   (b) Determine the difference in the amount of current flowing through the circuit from the fourth to the eighth seconds. Give your answer correct to 3 decimal places. [3 marks]

   (c) If current is allowed to flow through the circuit for a very long period of time, estimate $I(t)$. [2 marks]

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9.   In a delivery of microchips, it is known that the number of defective is 2 out of 10.

(a)   If 15 microchips are delivered, calculate the probability that

(i)   at least 5 microchips are defective.   [3 marks]

(ii)   exactly 11 microchips are good.   [2 marks]

(b)   If 500 microchips are delivered, find n such that the probability of obtaining the number of defective microchips exceeding n is 0.147.   [5 marks]

(c)   Suppose in another shipment of microchips, the probability of defective is 0.01. If a sample of 300 microchips is taken from the shipment, estimate the probability of getting 1 to 3 defective microchips.   [3 marks]

10.   The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} kx^2, & -1 < x < 2 \\ k, & 2 \le x < 4 \\ 0, & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a)   Find k.   [3 marks]

(b)   Determine , $F(X)$, the cumulative distribution function for X.   [4 marks]

(c)   Calculate

(i)   Var(X).   [4 marks]

(ii)   Var($3X - 2$).   [1 mark]

(iii) $P\big(|X| > 1\big)$.   [3 marks]

END OF QUESTION PAPER

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