QS026/1
Mathematics
Paper 1
Semester II
Session 2004/2005
2 hours

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Matematik
Kertas 1
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KEMENTERIAN PELAJARAN MALAYSIA

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.
This booklet consists of **11** *printed pages.*

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** the questions.

The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.

Only non–programmable scientific calculators can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropiate, unless stated otherwise in the question.

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LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \operatorname{kos} B \pm \operatorname{kos} A \sin B$$

$$\operatorname{kos}(A \pm B) = \operatorname{kos} A \operatorname{kos} B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Differentiation and Integration

$f(x)$	$f'(x)$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Coordinate Geometry

Perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Trapezium Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$

Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots$$

Sphere	$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$
Right circular cone	$V = \frac{1}{3}\pi r^2 h$	$S = \pi r s$
Right circular cylinder	$V = \pi r^2 h$	$S = 2\pi rh$

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1. Show that $\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}=2\sec^2\theta.$ [2 marks]

   Hence, evaluate $\displaystyle\int_0^{\frac{\pi}{4}}\left(\frac{1}{1-\sin 3x}+\frac{1}{1+\sin 3x}\right)dx.$ [4 marks]

2. Use the trapezoidal rule to approximate $\displaystyle\int_{-1}^{1}\sqrt{1-x^2}\,dx$ with 6 subintervals, giving your answer correct to three decimal places. [6 marks]

3. Solve the following differential equation,

$$\frac{dy}{dx}=xe^{x-2y};\quad y(0)=1.$$ [6 marks]

4. Find $\dfrac{dy}{dx}$ if

   (a) $y=\tan^3(x^3+2)$. [3 marks]

   (b) $\sin(x-y)=y\cos x$. [4 marks]

5. Find the vertices, foci and the equation of asymptotes of the hyperbola
   $9x^2-16y^2+54x+64y-127=0.$ [9 marks]
   Sketch the hyperbola and label the vertices, foci and its asymptotes. [3 marks]

6. (a) Given that $f(x)=\cos x,\ 0\le x\le\pi$. State the domain and range of $f^{-1}(x)$. Sketch the graphs of f and f^{-1} on the same coordinate axes. [6 marks]

   (b) If $\tan\left(\dfrac{x}{2}\right)=t$, find $\sin x$ and $\cos x$ in terms of t. Hence, solve
   $\cos x+7\sin x=5$, for $0\le x\le\pi$.

   [6 marks]

7. Given that $f(x) = 3x^4 - 4x^3 + 1$.

(a) Find the intervals of x where $f(x)$ is increasing and decreasing. [4 marks]

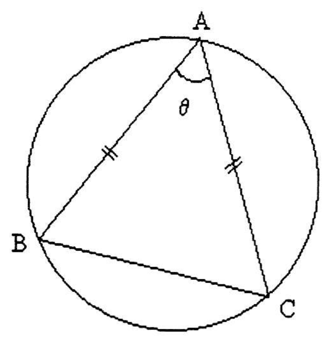
(b) Use the first derivative test to determine the relative maximum or minimum (if any). [3 marks]

(c) Find the intervals of x where the graph $f(x)$ is concave up and concave down. Hence, find the inflection points (if any). [5 marks]

8. (a) Find the foci of $9x^2 + 4y^2 = 36$ and sketch its graph. [5 marks]

(b) By using implicit differentiation, find the gradient of the tangent to the curve $9x^2 + 4y^2 = 36$. Hence, find the coordinates on the curve with gradient $\frac{9}{2}$. [7 marks]

9. The figure below shows a triangle ABC circumscribed in a circle of radius r. The sides AB and AC are equal in length and the angle BAC is θ.



(a) Prove that AB = $2r\cos\frac{\theta}{2}$. Hence, if L is the area of the triangle ABC, show that $L = r^2(1+\cos\theta)\sin\theta$. [4 marks]

(b) Show that $\frac{d^2L}{d\theta^2} = -r^2(\sin\theta + 2\sin 2\theta)$. [3 marks]

(c) If the value of θ varies, find the maximum area of the triangle in terms of r. [5 marks]

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10. A(6, 3, 3), B(3, 5, 1) and C(−1, 3, 5) are points in a three-dimensional space. Find

(a) the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in terms of unit vectors **i**, **j** dan **k**. Hence, show that $\overrightarrow{BA}$ is perpendicular to $\overrightarrow{BC}$, [6 marks]

(b) a unit vector that is perpendicular to the plane containing the points A, B and C, [6 marks]

(c) a Cartesian equation of the plane described in (b). [3 marks]

END OF QUESTION PAPER

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