

QS 025/1
Matriculation Programme
Examination
Semester I
Session 2014/2015

1. Use the trapezoidal rule to estimate $\int_0^1 f(x) dx$ from the data given below:

x	0.00	0.25	0.5	0.75	1.00
$f(x)$	2.4	2.6	2.9	3.2	3.6

2. Given a parabola with vertex $(-2, 1)$, opening to the right and passes through the point $(3, 6)$. Find the equation of the parabola and determine its focus.
3. Evaluate the following integrals:
- $\int \sin 6x \cos 4x dx$
 - $\int (3 \tan x + 4)^5 \sec^2 x dx$
4. Use the Newton-Raphson method with initial approximation $x_1 = 1$ to find $\sqrt[6]{2}$ on $[0, 2]$ correct to three decimal places.
5. Find the equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which passes through the points $A(0, 1)$, $B(3, -2)$ and $C(-1, -4)$. Hence, determine its center and radius. Find the points of intersection of the circle with the y-axis.
6. Given that $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{x}{4}$.
- On the same axes, sketch the graphs of f and g for the values of x between $x = 0$ and $x = 2$. Shade the region R bounded by f , g , $x = 0$ and $x = 2$.
 - Find the area of region R .
 - Find the volume of the solid generated when the region R is rotated through 2π radian about the x-axis.
7. (a) The amount $Q(t)$ of radioactive substance present at time t in a reaction is given by the differential equation
- $$\frac{dQ}{dt} = -kQ$$
- where k is a positive constant. If the initial amount of the substance is 100mg and is decrease to 97 mg in 6 days, determine
- The half-life of the substance
 - The amount of radioactive substance present after 30 days.
- b. Find the general solution to the differential equation
- $$(1 + x) \frac{dy}{dx} - y = 1 + x.$$
8. Given two straight lines,

$$L_1: t = \frac{x-1}{-3} = \frac{y+2}{8} = \frac{z}{-3} \quad \text{and} \quad L_2: t = \frac{x+2}{10} = \frac{y}{10} = \frac{z-4}{-7}.$$

- a. Show that L_1 and L_2 are not parallel and find the acute angle between the two straight lines.
- b. Determine intersection point between L_1 and plane $\Pi: 2x - y + 5z + 25 = 0$.
- c. Find an equation of the plane containing L_1 and L_2 .
9. (a) Find the values of A, B, C and D if $\frac{x^2+9}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-3)}$.
- (b) Hence, evaluate $\int_{-2}^1 \frac{x^2+9}{x^2(x-3)} dx$.
10. Given P, Q and R are three points in a space where
- $$\overrightarrow{PQ} = \mathbf{a} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \overrightarrow{PR} = \mathbf{b} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$$
- and the coordinates of R is (3, 0, 1).
- a. Hence, show that
- \mathbf{a} and \mathbf{b} are not perpendicular.
 - $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$
- b. Find the area of triangle PQR.
- c. Find the Cartesian equation for the
- Plane that passes through the points P, Q and R.
 - Line that passes through the point R and perpendicular to the plane in part (i).

END OF QUESTION PAPER

1. Use the trapezoidal rule to estimate $\int_0^1 f(x) dx$ from the data given below:

x	0.00	0.25	0.5	0.75	1.00
$f(x)$	2.4	2.6	2.9	3.2	3.6

SOLUTION

$$h = 0.25 - 0.00 = 0.25$$

x	$f(x)$		
$x_0 = 0.00$	y_0	2.4	
$x_1 = 0.25$	y_1		2.6
$x_2 = 0.50$	y_2		2.9
$x_3 = 0.75$	y_3		3.2
$x_4 = 1.00$	y_4	3.6	
Total		6.0	8.7

$$\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$

$$\begin{aligned} \int_0^1 f(x) dx &= \frac{0.25}{2} [6.0 + 2(8.7)] \\ &= 2.925 \end{aligned}$$

2. Given a parabola with vertex $(-2, 1)$, opening to the right and passes through the point $(3, 6)$.
Find the equation of the parabola and determine its focus.

SOLUTION

Open to the right

$$(y - k)^2 = 4p(x - h)$$

$$V(h, k) = V(-2, 1)$$

$$h = -2, k = 1$$

$$(y - 1)^2 = 4p(x - (-2))$$

$$(y - 1)^2 = 4p(x + 2)$$

At $(3, 6)$:

$$(6 - 1)^2 = 4p(3 + 2)$$

$$25 = 20p$$

$$p = \frac{5}{4}$$

$$(y - 1)^2 = 4\left(\frac{5}{4}\right)(x + 2)$$

$$(y - 1)^2 = 5(x + 2)$$

$$\begin{aligned} F(h + p, k) &= F\left(-2 + \frac{5}{4}, 1\right) \\ &= F\left(-\frac{3}{4}, 1\right) \end{aligned}$$

3. Evaluate the following integrals:

a. $\int \sin 6x \cos 4x \, dx$

b. $\int (3 \tan x + 4)^5 \sec^2 x \, dx$

SOLUTION

a)

$$\begin{aligned}\int \sin 6x \cos 4x \, dx &= \int \frac{1}{2} [\sin(6x + 4x) + \sin(6x - 4x)] \, dx \\ &= \frac{1}{2} \int \sin 10x + \sin 2x \, dx \\ &= \frac{1}{2} \left(-\frac{\cos 10x}{10} - \frac{\cos 2x}{2} \right) + C \\ &= -\frac{1}{20} \cos 10x - \frac{1}{4} \cos 2x + C\end{aligned}$$

b) $\int (3 \tan x + 4)^5 \sec^2 x \, dx = \int u^5 \left(\frac{du}{3} \right)$

$$\begin{aligned}&= \frac{1}{3} \int u^5 \, du \\ &= \frac{1}{3} \left(\frac{u^6}{6} \right) + C \\ &= \frac{1}{18} (3 \tan x + 4)^6 + C\end{aligned}$$

$u = 3 \tan x + 4$
$du = 3 \sec^2 x \, dx$

4. Use the Newton-Raphson method with initial approximation $x_1 = 1$ to find $\sqrt[6]{2}$ on $[0, 2]$ correct to three decimal places.

SOLUTION

$$\text{Let } x = \sqrt[6]{2}$$

$$x^6 = 2$$

$$x^6 - 2 = 0$$

$$f(x) = x^6 - 2$$

$$f'(x) = 6x^5$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^6 - 2)}{6x_n^5}$$

$$x_1 = 1$$

$$x_2 = 1 - \frac{[(1)^6 - 2]}{6(1)^5} = 1.1667$$

$$x_3 = 1.1667 - \frac{[(1.1667)^6 - 2]}{6(1.1667)^5} = 1.1264$$

$$x_4 = 1.1264 - \frac{[(1.1264)^6 - 2]}{6(1.1264)^5} = 1.1225$$

$$x_5 = 1.1225 - \frac{[(1.1225)^6 - 2]}{6(1.1225)^5} = 1.1225$$

$$x = 1.123$$

5. Find the equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which passes through the points $A(0, 1)$, $B(3, -2)$ and $C(-1, -4)$. Hence, determine its center and radius. Find the points of intersection of the circle with the y-axis.

SOLUTION

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\begin{aligned} (0, 1) : (0)^2 + (1)^2 + 2g(0) + 2f(1) + c &= 0 \\ 1 + 2f + c &= 0 \\ 2f + c &= -1 \quad \dots\dots\dots (1) \end{aligned}$$

$$\begin{aligned} (3, -2) : (3)^2 + (-2)^2 + 2g(3) + 2f(-2) + c &= 0 \\ 13 + 6g - 4f + c &= 0 \\ 6g - 4f + c &= -13 \quad \dots\dots\dots (2) \end{aligned}$$

$$\begin{aligned} (-1, -4) : (-1)^2 + (-4)^2 + 2g(-1) + 2f(-4) + c &= 0 \\ 17 - 2g - 8f + c &= 0 \\ -2g - 8f + c &= -17 \quad \dots\dots\dots (3) \end{aligned}$$

$$\begin{aligned} (2) - (1) : 6g - 6f &= -12 \\ g - f &= -2 \quad \dots\dots\dots (4) \end{aligned}$$

$$\begin{aligned} (3) - (1) : -2g - 10f &= -16 \\ -g - 5f &= -8 \quad \dots\dots\dots (5) \end{aligned}$$

$$\begin{aligned} (4) + (5) : -6f &= -10 \\ f &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} (4) : g - \frac{5}{3} &= -2 \\ g &= -\frac{1}{3} \end{aligned}$$

$$\begin{aligned} (1) : 2\left(\frac{5}{3}\right) + c &= -1 \\ c &= -\frac{13}{3} \end{aligned}$$

$$x^2 + y^2 + 2\left(-\frac{1}{3}\right)x + 2\left(\frac{5}{3}\right)y + \left(-\frac{13}{3}\right) = 0$$

$$3x^2 + 3y^2 - 2x + 10y - 13 = 0$$

$$C(-g, -f) = C\left(\frac{1}{3}, -\frac{5}{3}\right)$$

$$\boxed{r = \sqrt{g^2 + f^2 - c}}$$

$$r = \sqrt{\left(-\frac{1}{3}\right)^2 + \left(\frac{5}{3}\right)^2 - \left(-\frac{13}{3}\right)} = \sqrt{\frac{65}{9}} = \frac{\sqrt{65}}{3}$$

At y-axis, $x = 0$

$$3(0)^2 + 3y^2 - 2(0) + 10y - 13 = 0$$

$$3y^2 + 10y - 13 = 0$$

$$(3y + 13)(y - 1) = 0$$

$$y = -\frac{13}{3} \quad \text{or} \quad y = 1$$

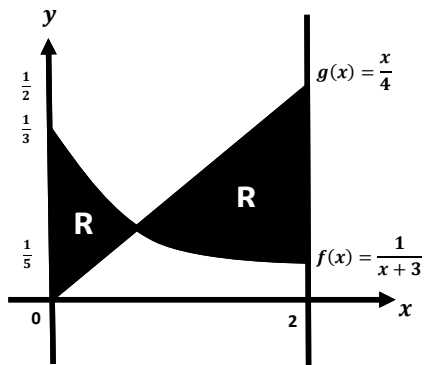
The points of intersection are $\left(0, -\frac{13}{3}\right)$ and $(0, 1)$

6. Given that $f(x) = \frac{1}{x+3}$ and $g(x) = \frac{x}{4}$.
- On the same axes, sketch the graphs of f and g for the values of x between $x = 0$ and $x = 2$. Shade the region R bounded by f , g , $x = 0$ and $x = 2$.
 - Find the area of region R .
 - Find the volume of the solid generated when the region R is rotated through 2π radian about the x -axis.

SOLUTION

$$f(x) = \frac{1}{x+3}, \quad g(x) = \frac{x}{4}$$

a)



- b) $\frac{x}{4} = \frac{1}{x+3}$
 $x^2 + 3x = 4$
 $x^2 + 3x - 4 = 0$
 $(x+4)(x-1) = 0$
 $x = -4$ or $x = 1$
 From graph, $x = 1$

$$A = \int_{x_1}^{x_2} y \, dx$$

$$\begin{aligned} \text{Area} &= \int_0^1 \frac{1}{x+3} - \frac{x}{4} \, dx + \int_1^2 \frac{x}{4} - \frac{1}{x+3} \, dx \\ &= \left[\ln(x+3) - \frac{x^2}{8} \right]_0^1 + \left[\frac{x^2}{8} - \ln(x+3) \right]_1^2 \\ &= \left[\ln 4 - \frac{(1)^2}{8} \right] - [\ln 3 - 0] + \left[\frac{(2)^2}{8} - \ln 5 \right] - \left[\frac{(1)^2}{8} - \ln 4 \right] \\ &= 0.315 \text{ unit}^2 \end{aligned}$$

c)

$$V = \pi \int_{x_1}^{x_2} y^2 dx$$

$$\text{Volume} = \pi \int_0^1 \left(\frac{1}{x+3} \right)^2 - \left(\frac{x}{4} \right)^2 dx + \pi \int_1^2 \left(\frac{x}{4} \right)^2 - \left(\frac{1}{x+3} \right)^2 dx$$

$$= \pi \int_0^1 \frac{1}{(x+3)^2} - \frac{x^2}{16} dx + \pi \int_1^2 \frac{x^2}{16} - \frac{1}{(x+3)^2} dx$$

$$= \pi \left[-\frac{1}{x+3} - \frac{x^3}{48} \right]_0^1 + \pi \left[\frac{x^3}{48} + \frac{1}{x+3} \right]_1^2$$

$$= \pi \left[\left(-\frac{1}{4} - \frac{1}{48} \right) - \left(-\frac{1}{3} - 0 \right) \right] + \pi \left[\left(\frac{(2)^3}{48} + \frac{1}{5} \right) - \left(\frac{1}{48} + \frac{1}{4} \right) \right]$$

$$= \frac{19}{120} \pi \text{ unit}^3$$

$$\begin{aligned} \int -\frac{1}{(x+3)^2} dx &= \int -(x+3)^{-2} dx \\ &= -\left(\frac{(x+3)^{-1}}{-1} \right) \\ &= \frac{1}{x+3} \end{aligned}$$

7. (a) The amount $Q(t)$ of radioactive substance present at time t in a reaction is given by the differential equation

$$\frac{dQ}{dt} = -kQ$$

where k is a positive constant. If the initial amount of the substance is 100mg and is decrease to 97 mg in 6 days, determine

- i. The half-life of the substance
 - ii. The amount of radioactive substance present after 30 days.
- b. Find the general solution to the differential equation

$$(1 + x) \frac{dy}{dx} - y = 1 + x.$$

SOLUTION

a) $\frac{dQ}{dt} = -kQ$

i) $\int \frac{1}{Q} dQ = \int -k dt$

$$\ln Q = -kt + C$$

$$Q = e^{-kt+C}$$

$$Q = e^{kt} e^C$$

$$Q = Ae^{-kt} \quad \text{where } A = e^C$$

$$t = 0, \quad Q = 100$$

$$100 = Ae^{-k(0)}$$

$$A = 100$$

$$Q = 100 e^{-kt}$$

$$t = 6, \quad Q = 97$$

$$97 = 100 e^{-k(6)}$$

$$0.97 = e^{-6k}$$

$$\ln(0.97) = -6k$$

$$k = 0.00508$$

$$Q = 100 e^{-0.00508t}$$

$$50 = 100 e^{-0.00508t}$$

$$0.5 = e^{-0.00508t}$$

$$\ln(0.5) = -0.00508t$$

$$t = 136 \text{ days}$$

ii) $Q = 100 e^{-0.00508(30)}$

$$Q = 85.86 \text{ mg}$$

$$\text{b) } (1+x) \frac{dy}{dx} - y = 1+x$$

$$\frac{dy}{dx} - \frac{1}{1+x} y = 1$$

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$$

$$P(x) = -\frac{1}{1+x}, \quad Q(x) = 1$$

$$\boxed{V(x) = e^{\int P(x) dx}}$$

$$\begin{aligned} V(x) &= e^{\int -\frac{1}{1+x} dx} \\ &= e^{-\ln(1+x)} \\ &= e^{\ln(1+x)^{-1}} \\ &= (1+x)^{-1} \\ &= \frac{1}{1+x} \end{aligned}$$

$$\boxed{V(x)y = \int V(x)Q(x) dx}$$

$$\begin{aligned} \left(\frac{1}{1+x}\right)y &= \int \left(\frac{1}{1+x}\right)(1) dx \\ &= \int \frac{1}{1+x} dx \end{aligned}$$

$$\frac{y}{1+x} = \ln(1+x) + C$$

$$y = (1+x)(\ln(1+x) + C)$$

8. Given two straight lines,

$$L_1: t = \frac{x-1}{-3} = \frac{y+2}{8} = \frac{z}{-3} \quad \text{and} \quad L_2: t = \frac{x+2}{10} = \frac{y}{10} = \frac{z-4}{-7}.$$

- Show that L_1 and L_2 are not parallel and find the acute angle between the two straight lines.
- Determine intersection point between L_1 and plane $\Pi: 2x - y + 5z + 25 = 0$.
- Find an equation of the plane containing L_1 and L_2 .

SOLUTION

$$L_1: t = \frac{x-1}{-3} = \frac{y+2}{8} = \frac{z}{-3} \quad \text{and} \quad L_2: t = \frac{x+2}{10} = \frac{y}{10} = \frac{z-4}{-7}$$

a) $\underline{v}_1 = -3\underline{i} + 8\underline{j} - 3\underline{k}$

$$\underline{v}_2 = 10\underline{i} + 10\underline{j} - 7\underline{k}$$

$$\underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3 & 8 & -3 \\ 10 & 10 & -7 \end{vmatrix}$$

$$= (-56 + 30)\underline{i} - (21 + 30)\underline{j} + (-30 - 80)\underline{k}$$

$$= -26\underline{i} - 51\underline{j} - 110\underline{k}$$

$$\neq \underline{0}$$

L_1 and L_2 are not parallel

$$\underline{a} \text{ and } \underline{b} \text{ are parallel} \\ \Rightarrow \underline{a} \times \underline{b} = \underline{0}$$

$$\underline{v}_1 \cdot \underline{v}_2 = (-3\underline{i} + 8\underline{j} - 3\underline{k}) \cdot (10\underline{i} + 10\underline{j} - 7\underline{k})$$

$$= -30 + 80 + 21$$

$$= 71$$

$$|\underline{v}_1| = \sqrt{(-3)^2 + 8^2 + (-3)^2} = \sqrt{82}$$

$$|\underline{v}_2| = \sqrt{10^2 + 10^2 + (-7)^2} = \sqrt{249}$$

$$\underline{v}_1 \cdot \underline{v}_2 = |\underline{v}_1| |\underline{v}_2| \cos \theta$$

$$71 = (\sqrt{82})(\sqrt{249}) \cos \theta$$

$$\theta = 60.2^\circ$$

b) $\pi: 2x - y + 5z + 25 = 0$

Parametric equation of line L_1

$$x = 1 - 3t, \quad y = -2 + 8t, \quad z = -3t$$

substitute equation of line into equation of plane

$$2(1 - 3t) - (-2 + 8t) + 5(-3t) + 25 = 0$$

$$2 - 6t + 2 - 8t - 15t + 25 = 0$$

$$29 - 29t = 0$$

$$-29t = -29$$

$$t = 1$$

$$x = 1 - 3(1) = -2$$

$$y = -2 + 8(1) = 6$$

$$z = -3(1) = -3$$

The intersection point is $(-2, 6, -3)$

c) $\underline{n} = \underline{v}_1 \times \underline{v}_2$

$$= -26\underline{i} - 51\underline{j} - 110\underline{k}$$

$$\underline{a} = \underline{i} - 2\underline{j}$$

Equation of plane

$$\underline{r} \cdot \underline{n} = \underline{a} \cdot \underline{n}$$

$$-26x - 51y - 110z = -26(1) - 51(-2) - 110(0)$$

$$-26x - 51y - 110z = 76$$

$$26x + 51y + 110z = -76$$

9. (a) Find the values of A, B, and C if $\frac{x^2+9}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-3)}$.

(b) Hence, evaluate $\int_{-2}^1 \frac{x^2+9}{x^2(x-3)} dx$.

SOLUTION

$$\begin{aligned} \text{a) } \frac{x^2+9}{x^2(x-3)} &= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-3)} \\ x^2+9 &= Ax(x-3) + B(x-3) + Cx^2 \\ x=0 : 9 &= B(-3) \\ B &= -3 \end{aligned}$$

$$\begin{aligned} x=3 : (3)^2+9 &= C(3)^2 \\ 18 &= 9C \\ C &= 2 \end{aligned}$$

$$\begin{aligned} x^2 : 1 &= A+C \\ 1 &= A+2 \\ A &= -1 \end{aligned}$$

$$\frac{x^2+9}{x^2(x-3)} = -\frac{1}{x} - \frac{3}{x^2} + \frac{2}{x-3}$$

$$\begin{aligned} \text{b) } \int_{-2}^1 \frac{x^2+9}{x^2(x-3)} dx &= \int_{-2}^1 -\frac{1}{x} - \frac{3}{x^2} + \frac{2}{x-3} dx \\ &= \left[-\ln|x| + \frac{3}{x} + 2\ln|x-3| \right]_{-2}^1 \\ &= \left[-\ln|1| + \frac{3}{1} + 2\ln|-2| \right] - \left[-\ln|-2| + \frac{3}{(-2)} + 2\ln|-5| \right] \\ &= 3.36 \end{aligned}$$

10. Given P, Q and R are three points in a space where

$$\overrightarrow{PQ} = \mathbf{a} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \overrightarrow{PR} = \mathbf{b} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$$

and the coordinates of R is (3, 0, 1).

- a. Hence, show that
 - i. \mathbf{a} and \mathbf{b} are not perpendicular.
 - ii. $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$
- b. Find the area of triangle PQR.
- c. Find the Cartesian equation for the
 - i. Plane that passes through the points P, Q and R.
 - ii. Line that passes through the point R and perpendicular to the plane in part (i).

SOLUTION

a) $\overrightarrow{PQ} = \underline{a} = 3\underline{i} - \underline{j} + \underline{k}$, $\overrightarrow{PR} = \underline{b} = 2\underline{i} + \underline{j} - 3\underline{k}$, R(3, 0, 1)

$$\begin{aligned} \text{i) } \underline{a} \cdot \underline{b} &= (3\underline{i} - \underline{j} + \underline{k}) \cdot (2\underline{i} + \underline{j} - 3\underline{k}) \\ &= 6 - 1 - 3 \\ &= 2 \\ &\neq 0 \end{aligned}$$

\underline{a} and \underline{b} are not perpendicular

$$\begin{aligned} \underline{a} \text{ and } \underline{b} \text{ are perpendicular} \\ \Rightarrow \underline{a} \cdot \underline{b} = 0 \end{aligned}$$

$$\begin{aligned} \text{ii) } \underline{a} \times \underline{b} &= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix} \\ &= (3-1)\underline{i} - (-9-2)\underline{j} + (3+2)\underline{k} \\ &= 2\underline{i} + 11\underline{j} + 5\underline{k} \end{aligned}$$

$$|\underline{a} \times \underline{b}| = \sqrt{2^2 + 11^2 + 5^2} = \sqrt{150}$$

$$|\underline{a} \times \underline{b}|^2 = 150$$

$$|\underline{a}| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{11}$$

$$|\underline{a}|^2 = 11$$

$$|\underline{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{14}$$

$$|\underline{b}|^2 = 14$$

$$\underline{a} \cdot \underline{b} = 2$$

$$(\underline{a} \cdot \underline{b})^2 = 2^2 = 4$$

$$|\underline{a}|^2 |\underline{b}|^2 - (\underline{a} \cdot \underline{b})^2 = (11)(14) - (4) = 150$$

$$|\underline{a} \times \underline{b}|^2 = |\underline{a}|^2 |\underline{b}|^2 - (\underline{a} \cdot \underline{b})^2$$

$$\text{b) } \overrightarrow{PQ} \times \overrightarrow{PR} = \underline{a} \times \underline{b} = 2\underline{i} + 11\underline{j} + 5\underline{k}$$

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = |\underline{a} \times \underline{b}| = \sqrt{150}$$

$$\begin{aligned} \text{Area of triangle PQR} &= \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| \\ &= \frac{1}{2} \sqrt{150} \\ &= 6.124 \text{ unit}^2 \end{aligned}$$

$$\text{c) i) } \underline{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = 2\underline{i} + 11\underline{j} + 5\underline{k}$$

$$\text{R}(3, 0, 1)$$

Cartesian equation of plane

$$2x + 11y + 5z = 2(3) + 11(0) + 5(1)$$

$$2x + 11y + 5z = 11$$

$$\text{ii) } \underline{v} = \underline{n} = 2\underline{i} + 11\underline{j} + 5\underline{k}$$

$$\underline{a}_1 = 3\underline{i} + \underline{k}$$

Cartesian equation of line

$$\frac{x-3}{2} = \frac{y}{11} = \frac{z-1}{5}$$