KEMENTERIAN PENDIDIKAN MALAYSIA

QS 025/2

Matriculation Programme Examination

Semester II

Session 2015/2016

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1. Weights (kg) of a random sample of 90 female students and 105 male student is summarised as $\sum(x-50)=-234$ and $\sum(y-63)=367.5$ respectively. Calculate the mean weight of all the students.

2. The length of newborn babies at a hospital for a particular year is normally distributed with mean of 52 cm and standard deviation of 2.5 cm. A baby's length is considered normal if it is between 46cm and 56 cm. From a list of 100 birth records selected randomly for that particular year at the hospital, how many babies are expected to have normal lengths?

3. A car rental company has 7 cars available for rental each day. Assuming that each rental is for the whole day and that the number of demands has a mean of 3 cars per day. Find the probability that
   a) The company cannot meet the demand in any one day.
   b) Less than 5 cars are rented in a period of 3 days.

4. Given $P(A)=0.37, P(B|A)=0.13\ \ and\ P(A'\cap B)=0.47$. Find
   a) $P(A\cap B)$.
   b) $P(B)$ and hence calculate $P(A\cup B)'$

5. Given a set of digits $\{0,1,2,3,4,5,6,7,8,9\}$.
   a) Find the number of different ways to choose two prime digits from the set.
   b) Four-digit numbers are to be formed from the set and the numbers do not start with digit 0. Find the possible number of ways of getting
      i. Even numbers between 6000 and 7000 if every digit can be repeated.

ii. Numbers greater than 6000 that end with digit 5 and the digits can only be used once.

iii. Numbers that contain exactly two odd digits and they must be next to each other with no repetitions of digits allowed.

6. The time taken for 70 students to walk from the hostel to class in a certain college are shown in the following table.

Time (minute)	Number of student
2 – 4	5
5 – 7	9
8 – 10	19
11 – 13	21
14 – 16	12
17 - 19	4

a) Find the mean and mode.

b) Determine the 40^{th} percentile.

c) Find the standard deviation.

d) Calculate the Pearson's coefficient of skewness. Interpret your answer.

7. A car insurance company offers two types of insurance plan for privately-owned cars, namely Plan X and Plan Y. For a random sample of 60 clients for each insurance plan, the number of claims is given in the following table.

Plan	Claim	
	Yes	No
X	38	22
Y	45	15

Let

A = the event that no claim is made by the client

B = the event that the customer takes Plan X.

a) Find $P(A \cap B)$.

b) Find $P(A' \cap B)$

c) Given that the chosen client did not make any claim, find the probability that the insurance plan taken was Plan X.

d) Determine whether the events “make a claim” and “the type of each insurance plan taken” are independent. Give reason for your answer.

8. It is known that 37% of the students at a college do not take breakfast regularly. A random sample of 20 students is chosen.

a) Find the probability that there are at least two students who do not take breakfast regularly.

b) Use normal approximation to calculate the probability that there are more than 10 students who do not take breakfast regularly. Verify that the distribution can be approximated by a normal distribution.

9. The continuous random variable X has cumulative distribution function F(x) given by

$$F(x)=\begin{cases} 0 & , \quad x \le 0 \\ \dfrac{x^2}{6} & , \quad 0 \le x \le 2 \\ -\dfrac{x^2}{3}+2x-2 & , \quad 2 \le x \le 3 \\ 1 & , \quad x \ge 3 \end{cases}$$

Find

a) $P(1 < X < 2.2)$

b) *the value of median*

c) The probability density function of X.

d) The expected value of X.

e) The variance of X, given that $E(x^2) = \frac{19}{6}$.

10. Two dice are thrown and the numbers x and y obtained from each dice are noted. The discrete random variable W is defined as

$$W = f(x) = \begin{cases} xy, & x = y \\ |x-y|, & x \ne y \end{cases}$$

a) Write all the outcomes for W=4 and hence show that

$$P(W=4) = \frac{5}{36}$$

b) Construct a table of the probability distribution of the random variable W. Hence, show that W is a discrete random variable.

c) Find $P(W > 9)$.

d) Find the mode of W.

e) Find $E(W)$ and hence, calculate $E(3 - 4W)$.

END OF QUESTION PAPER

1. Weights (kg) of a random sample of 90 female students and 105 male student is summarised as $\sum(x-50)=-234$ and $\sum(y-63)=367.5$ respectively. Calculate the mean weight of all the students.

SOLUTION

$$\sum(x-50)=-234$$

$$\sum_{n=1}^{90}(x_n-50)=-234$$

$(x_1-50)+(x_2-50)+(x_3-50)+\cdots+(x_{89}-50)$+$(x_{90}-50)$ = -234

$(x_1+x_2+x_3+\cdots+x_{89}+x_{90})+(-50)\ (90)$ = -234

$$\sum x-4500=-234$$

$$\sum x=-234+4500$$

$\sum x=4266$ .... (1)

$$\sum_{n=1}^{105}(x_n-63)=367.5$$

$(y_1-63)+(y-63)+(y_3-63)+\cdots+(y_{104}-63)$+$(y_{105}-63)$ = 367.5

$(y_1+y_2+y_3+\cdots+x_{104}+x_{105})+(-63)\ (105)$ = 367.5

$$\sum y-6615=367.5$$

$\sum y=6982.5$ .... (2)

$$Mean\ weight\ of\ all\ students=\frac{\sum x+\sum y}{n}$$

$$=\frac{4266+6982.5}{90+105}$$

$=$ 57.68

2. The length of newborn babies at a hospital for a particular year is normally distributed with mean of 52 cm and standard deviation of 2.5 cm. A baby's length is considered normal if it is between 46cm and 56 cm. From a list of 100 birth records selected randomly for that particular year at the hospital, how many babies are expected to have normal lengths?

SOLUTION

$X - body\ length\ of\ baby$

$\mu = 52, \qquad \sigma = 2.5$

$X \sim N(\mu, \sigma^2)$

$X \sim N(52, 2.5^2)$

$X \sim N(\mu, \sigma^2) \rightarrow Z \sim N(0,1)$

$$Z = \frac{X - \mu}{\mu}$$

$P(46 < X < 56)$

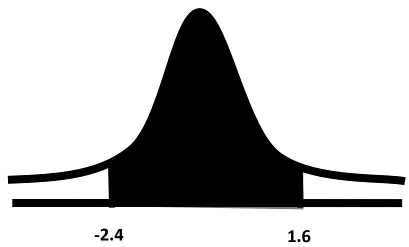
$= P\left(\frac{46-52}{2.5} < Z < \frac{56-52}{2.5}\right)$

$= P(-2.4 < Z < 1.6)$

$= 1 - P(Z > 1.6) - P(Z > 2.4)$

$= 1 - 0.0548 - 0.0082$

$= 0.937$



Thus, the expected number of normal length babies $= np$

$= 100\ x\ 0.937$

$= 93.7$

≈ 94

3. A car rental company has 7 cars available for rental each day. Assuming that each rental is for the whole day and that the number of demands has a mean of 3 cars per day. Find the probability that

   a) The company cannot meet the demand in any one day.

   b) Less than 5 cars are rented in a period of 3 days.

SOLUTION

(a) X = Number of cars rented in one day

$\lambda = 3$

$X \sim Po(3)$

$P(X > 7) = P(X \geq 8) = 0.0119$

(b) Y = number of cars rented in 3 days

$\lambda = 9$

$Y \sim Po(9)$

$P(X < 5) = 1 - P(X \geq 5)$

$= 1 - 0.9450$

$= 0.055$

4. Given $P(A) = 0.37, P(B|A) = 0.13 \ and \ P(A' \cap B) = 0.47$. Find

a) $P(A \cap B)$.

b) $P(B)$ and hence calculate $P(A \cup B)'$

SOLUTION

(a) $P(A) = 0.37$

$P(B|A) = 0.13$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A).P(B|A)$$

$$= (0.37).(0.13)$$

$$= 0.0481$$

(b) $P(A \cup B)' = 1 - [P(A) + P(B) - P(A \cap B)]$

$$P(A' \cap B) = P(B) - P(A \cap B)$$

$$P(B) = P(A' \cap B) + P(A \cap B)$$

$$= 0.47 + 0.0481$$

$$= 0.5181$$

$$P(A \cup B)' = 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [0.37 + 0.5181 - 0.0481]$$

$$= 0.16$$

5. Given a set of digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   a) Find the number of different ways to choose two prime digits from the set.

   b) Four-digit numbers are to be formed from the set and the numbers do not start with digit 0. Find the possible number of ways of getting

      i. Even numbers between 6000 and 7000 if every digit can be repeated.

      ii. Numbers greater than 6000 that end with digit 5 and the digits can only be used once.

      iii. Numbers that contain exactly two odd digits and they must be next to each other with no repetitions of digits allowed.

SOLUTION

(a) $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$P = \{\, 2, 3, 5, 7 \,\}$

The number of different ways to choose two prime digits from the set = $^4C_2 = 6$

(bi) Possible number of ways of getting Four-digit even numbers between 6000 and 7000 if every digit can be repeated.

1	x	10	x	10	x	5	-	1
$\{6\}$		$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$		$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$		$\{0, 2, 4, 6, 8\}$		6000

Possible Digits

Possible number of ways $= (1 \times 10 \times 10 \times 5) - 1$

$= 499$

(bii) Possible number of ways of getting Four-digit Numbers greater than 6000 that end with digit 5 and the digits can only be used once.

4 x 8 x 7 x 1

$\{6, 7, 8, 9\}$ $\{5\}$

Possible Digits

Possible number of ways = $(4\ x\ 8\ x\ 7\ x\ 1)$

= 224

(biii) Possible number of ways of getting Four-digit Numbers Numbers that contain exactly two odd digits and they must be next to each other with no repetitions of digits allowed.

O O E E	5 X 4 X 5 X 4 = 400
E O O E	4 X 5 X 4 X 4 = 320
E E O O	4 X 4 X 5 X 4 = 320
Total	1040

Possible number of ways = $(5\ x\ 4\ x\ 5\ x\ 4) + (4\ x\ 5\ x\ 4\ x\ 4) + (4\ x\ 4\ x\ 5\ x\ 4)$

= 1040

6. The time taken for 70 students to walk from the hostel to class in a certain college are shown in the following table.

Time (minute)	Number of student
2 – 4	5
5 – 7	9
8 – 10	19
11 – 13	21
14 – 16	12
17 - 19	4

a) Find the mean and mode.

b) Determine the 40th percentile.

c) Find the standard deviation.

d) Calculate the Pearson's coefficient of skewness. Interpret your answer.

SOLUTION

Time (minute)	Class Boundary	x	f	fx	fx^2
2 – 4	1.5 – 4.5	3	5	15	45
5 – 7	4.5 – 7.5	6	9	54	324
8 – 10	7.5 – 10.5	9	19	171	1539
11 – 13	10.5 – 13.5	12	21	252	3024
14 – 16	13.5 – 16.5	15	12	180	2700
17 - 19	16.5 – 19.5	18	4	72	1296
Total			$\sum f = 70$	$\sum fx = 744$	$\sum fx^2 = 8928$

(a) Mean, $\bar{x} = \frac{\sum fx}{\sum f}$

$$= \frac{744}{70} = 10.63$$

$$Mode = L_M + \left[\frac{d_1}{d_1 + d_2}\right]c$$

$$= 10.5 + \left[\frac{21 - 19}{(21 - 19) + (21 - 12)}\right](13.5 - 10.5)$$

$$= 10.5 + \left[\frac{2}{2 + 9}\right]3$$

$$= 11.05$$

Time (minute)	Class Boundary	x	f	fx	fx^2	F
2 – 4	1.5 – 4.5	3	5	15	45	5
5 – 7	4.5 – 7.5	6	9	54	324	14
8 – 10	7.5 – 10.5	9	19	171	1539	33
11 – 13	10.5 – 13.5	12	21	252	3024	54
14 – 16	13.5 – 16.5	15	12	180	2700	66
17 - 19	16.5 – 19.5	18	4	72	1296	70
Total			$\sum f = 70$	$\sum fx = 744$	$\sum fx^2 = 8928$	

(b) Persentile: $P_k = L_k + \left[\frac{\left(\frac{k}{100}\right)n - F_{k-1}}{f_k}\right]c$

$$P_{40} = L_{40} + \left[\frac{\left(\frac{40}{100}\right)70 - F_{40-1}}{f_{40}}\right]c$$

$$= 7.5 + \left[\frac{28 - 14}{19}\right](10.5 - 7.5)$$

$$= 9.71$$

(c) Variance: $s^2 = \frac{\sum fx^2 - \frac{1}{n}(\sum fx)^2}{n-1}$

$$s^2 = \frac{8928 - \frac{1}{70}(744)^2}{70 - 1}$$

$$= 14.788$$

Standard deviation: $s = \sqrt{s^2}$

$$= \sqrt{14.788}$$

$$= 3.85$$

(d) Pearson's coefficient of skewness $Sk = \frac{Mean - Mode}{Standard\ Deviation}$

$$= \frac{10.63 - 11.05}{3.85}$$

$$= -0.109$$

$\therefore Data\ is\ negatively\ skewed$

7. A car insurance company offers two types of insurance plan for privately-owned cars, namely Plan X and Plan Y. For a random sample of 60 clients for each insurance plan, the number of claims is given in the following table.

Plan	Claim	
	Yes	No
X	38	22
Y	45	15

Let
A = the event that no claim is made by the client
B = the event that the customer takes Plan X.

a) Find $P(A \cap B)$.
b) Find $P(A' \cap B)$
c) Given that the chosen client did not make any claim, find the probability that the insurance plan taken was Plan X.
d) Determine whether the events “make a claim” and “the type of each insurance plan taken” are independent. Give reason for your answer.

SOLUTION

Plan	Claim		Total
	Yes / (A’)	No / (A)	
X / (B)	38	22	60
Y / (B’)	45	15	60
Total	83	37	120

(a) $P(A \cap B) = \frac{22}{120}$

$= \frac{11}{60}$

(b) $P(A' \cap B) = P(A') + P(B) - P(A' \cap B)$

$= \frac{83}{120} + \frac{60}{120} - \frac{38}{120}$

$= \frac{105}{120} = \frac{7}{8}$

(c) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{22}{37}$

(d) $P(B) = \frac{1}{2}$

$P(B|A') = \frac{38}{83}$

$P(B|A') \neq P(B)$

$\therefore The\ event\ make\ a\ claim\ and\ type\ of\ insurance\ are\ not\ independent.$

8. It is known that 37% of the students at a college do not take breakfast regularly. A random sample of 20 students is chosen.

   a) Find the probability that there are at least two students who do not take breakfast regularly.
   b) Use normal approximation to calculate the probability that there are more than 10 students who do not take breakfast regularly. Verify that the distribution can be approximated by a normal distribution.

SOLUTION

(a) X – Students who do not take breakfast regularly

$X \sim B(20, 0.37)$

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - P(X \leq 0) - P(X = 1)$

$= 1 - {}^{20}C_0(0.37)^0\,(0.63)^{20} - {}^{20}C_1(0.37)^1\,(0.63)^{19}$

$= 0.9988$

(b) $\mu = np = 20\ x\ 0.37 = 7.4$

$\sigma^2 = npq = 20\ x\ 0.37\ x\ 0.63 = 4.662$

$X \sim N(7.4, 4.662)$

$P(X > 10)$

$= P(X > 10.5)$

Continuity Correction

$= P(Z > \frac{10.5 - 7.4}{\sqrt{4.662}})$

$Z = \frac{X - \mu}{\sigma}$

$= P(Z > \frac{10.5 - 7.4}{\sqrt{4.662}})$

$= P(Z > 1.44)$

$= 0.0749$

The distribution can be approximated by a normal distribution as

$np = 20\ x\ 0.37 = 7.4 > 5$

$nq = 20\ x\ 0.63 = 12.6 > 5$

9. The continuous random variable X has cumulative distribution function F(x) given by

$$F(x) = \begin{cases} 0 & , \quad x \leq 0 \\ \dfrac{x^2}{6} & , \quad 0 \leq x \leq 2 \\ -\dfrac{x^2}{3} + 2x - 2 & , \quad 2 \leq x \leq 3 \\ 1 & , \quad x \geq 3 \end{cases}$$

Find

a) $P(1 < X < 2.2)$

b) *the value of median*

c) The probability density function of X.

d) The expected value of X.

e) The variance of X, given that $E(x^2) = \frac{19}{6}$.

SOLUTION

(a) $P(1 < X < 2.2)$

$= F(2.2) - F(1)$

$= \left[-\dfrac{(2.2)^2}{3} + 2(2.2) - 2\right] - \left(\dfrac{1^2}{6}\right)$

$= 0.62$

(b) $F(m) = 0.5$

$\dfrac{m^2}{6} = 0.5$

$m^2 = 0.5\ x\ 6$

$m^2 = 3$

$m = \sqrt{3}$ since $m > 0$

(c)

	$F(x)$	$f(x) = \frac{d}{dx}F(x)$
$x \leq 0$	$F(x) = 0$	$f(x) = \frac{d}{dx}(0) = 0$
$0 \leq x \leq 2$	$F(x) = \frac{x^2}{6}$	$f(x) = \frac{d}{dx}\left(\frac{x^2}{6}\right) = \frac{x}{3}$
$2 \leq x \leq 3$	$F(x) = -\frac{x^2}{3} + 2x - 2$	$f(x) = \frac{d}{dx}\left(-\frac{x^2}{3} + 2x - 2\right) = -\frac{2x}{3} + 2$
$x \geq 3$	$F(x) = 1$	$f(x) = \frac{d}{dx}(0) = 0$

$$f(x)=\begin{cases}\frac{x}{3}\ , & 0\le x\le 2\\ -\frac{2x}{3}+2, & 2\le x\le 3\\ 0, & otherwise\end{cases}$$

(d) $E(x)=\int_{-\infty}^{\infty} x\,f(x)\,dx$

$$=\int_{-\infty}^{0} x\,(0)\,dx+\int_{0}^{2} x\left(\frac{x}{3}\right)dx+\int_{2}^{3} x\left(-\frac{2x}{3}+2\right)dx+\int_{3}^{\infty} x\,(0)\,dx$$

$$=\int_{0}^{2}\frac{x^2}{3}\,dx+\int_{2}^{3}-\frac{2x^2}{3}+2x\,dx$$

$$=\int_{0}^{2}\frac{x^2}{3}\,dx-\int_{2}^{3}\frac{2x^2}{3}-2x\,dx$$

$$=\left[\frac{x^3}{9}\right]_0^2-\left[\frac{2x^3}{9}-x^2\right]_2^3$$

$$=\left[\left(\frac{2^3}{9}\right)-\left(\frac{0^3}{9}\right)\right]-\left[\left(\frac{2(3)^3}{9}-(3)^2\right)-\left(\frac{2(2)^3}{9}-(2)^2\right)\right]$$

$$=\frac{8}{9}-\left[(6-9)-\left(\frac{16}{9}-4\right)\right]$$

$$=\frac{8}{9}-\left[-3-\left(-\frac{20}{9}\right)\right]$$

$$=\frac{5}{3}$$

(e) $var(x)=E(x^2)-[E(x)]^2$

$$=\frac{19}{6}-\left[\frac{5}{3}\right]^2$$

$$=\frac{7}{18}$$

10. Two dice are thrown and the numbers x and y obtained from each dice are noted. The discrete random variable W is defined as

$$W = f(x) = \begin{cases} xy, & x = y \\ |x - y|, & x \neq y \end{cases}$$

a) Write all the outcomes for W=4 and hence show that

$$P(W = 4) = \frac{5}{36}$$

b) Construct a table of the probability distribution of the random variable W. Hence, show that W is a discrete random variable.

c) Find $P(W > 9)$.

d) Find the mode of W.

e) Find $E(W)$ and hence, calculate $E(3 - 4W)$.

SOLUTION

y \ x	**1**	**2**	**3**	**4**	**5**	**6**
1	**1**	**1**	**2**	**3**	**4**	**5**
2	**1**	**4**	**1**	**2**	**3**	**4**
3	**2**	**1**	**9**	**1**	**2**	**3**
4	**3**	**2**	**1**	**16**	**1**	**2**
5	**4**	**3**	**2**	**1**	**25**	**1**
6	**5**	**4**	**3**	**2**	**1**	**36**

a) W=4

$\{(2,2), (5,1), (1,5), (2,6), (6,2)\}$

$$P(W = 4) = \frac{5}{36}$$

b)

W	1	2	3	4	5	9	16	25	36
P(W=w)	$\frac{11}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

$$\sum P(W=w)=\frac{11}{36}+\frac{8}{36}+\frac{6}{36}+\frac{5}{36}+\frac{2}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}=1$$

c) $P(W>9)=P(W=16)+P(W=25)+P(W=36)$

$$=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}$$

$$=\frac{3}{36}$$

$$=\frac{1}{12}$$

d) Mode = 1

e) $E(W)=\sum W[\mathrm{P(W=w)}]$

$$=1\left(\frac{11}{36}\right)+2\left(\frac{8}{36}\right)+3\left(\frac{6}{36}\right)+4\left(\frac{5}{36}\right)+5\left(\frac{2}{36}\right)+9\left(\frac{1}{36}\right)+16\left(\frac{1}{36}\right)+25\left(\frac{1}{36}\right)+36\left(\frac{1}{36}\right)$$

$$=\frac{161}{36}$$

f) $E(3-4W)=3-4E(W)$

$$=3-4\left(\frac{161}{36}\right)$$

$$=-\frac{134}{9}$$