QS015/2
Mathematics
Paper 2
Semester I
Session 2011/2012
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2011/2012
2 jam

BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
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PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **15** halaman bercetak.
This question paper consists of **15** *printed pages.*

INSTRUCTIONS TO CANDIDATE:

This question paper consists of **10** questions.

Answer **all** questions.

All answers must be written in the answer booklet provided. Use a new page for each question.

The full marks for each question or section are shown in the bracket at the end of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers may be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2 \sin\frac{A+B}{2} \cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos\frac{A+B}{2} \sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos\frac{A+B}{2} \cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin\frac{A+B}{2} \sin\frac{A-B}{2}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

LIST OF MATHEMATICAL FORMULAE

Limit

$$\lim_{h\to 0}\frac{\sin h}{h}=1$$

$$\lim_{h\to 0}\frac{1-\cos h}{h}=0$$

Differentiation

$f(x)$	$f'(x)$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\cot x$

If $y=g(t)$ and $x=f(t)$, then $\dfrac{dy}{dx}=\dfrac{dy}{dt}\times\dfrac{dt}{dx}$

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Sphere	$V=\frac{4}{3}\pi r^3$	$S=4\pi r^2$
Right circular cone	$V=\frac{1}{3}\pi r^2 h$	$S=\pi rs$
Right circular cylinder	$V=\pi r^2 h$	$S=2\pi rh$

KANG KOOI WEI

1 Express $\dfrac{6x-13}{(3x-4)^2}$ in the form of partial fractions.

[5 *marks*]

2 Evaluate the following limits:

(a) $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x-2}$.

[3 *marks*]

(b) $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x}}$.

[3 *marks*]

3 Find $\dfrac{dy}{dx}$ for the following equations:

(a) $y = 3^{2x+1}$.

[3 *marks*]

(b) $e^{xy} + y = 5x$.

[3 *marks*]

4 The surface area of a balloon in the shape of a sphere is decreasing at the rate of $2\ \text{cm}^2/\text{min}$. Find the rate at which the volume is decreasing when the radius of the balloon is 5 cm.

[7 *marks*]

5 (a) The function $f(x)=x^3-6x^2+9x-3$ is defined on the interval [0, 5]. Find the critical points of $f(x)$ on this interval and determine whether the critical points are local minimum or maximum.

[6 *marks*]

(b) Find the horizontal and vertical asymptotes for $f(x)=\dfrac{3x}{\sqrt{x^2-16}}$.

[7 *marks*]

6 The polynomial $p(x)=x^3-2x^2+ax+b$, where a and b are constants, has a factor of $(x-2)$ and leaves a remainder of a^3 when it is divided by $(x-a)$.

(a) Find the values of a and b.

[6 *marks*]

(b) Factorize $p(x)$ completely by using the values of a and b obtained from part **6**(a). Hence, find the real roots of $p(x)=0$, where a and b are not equal to zero.

[6 *marks*]

7 Given that $x=\dfrac{1}{\sqrt{1+t^2}}$ and $y=\dfrac{\sqrt{1+t^2}}{t}$, where t is a non zero parameter.

(a) Show that $\dfrac{dy}{dx}=\dfrac{1+t^2}{t^3}$.

[6 *marks*]

(b) Find $\dfrac{d^2y}{dx^2}$ when $t=1$.

[6 *marks*]

8 (a) If $y=\sin\left(x^2+1\right)$, show that

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}+4x^3y=0.$$

[5 *marks*]

(b) Find the gradient of a curve $xe^{xy}=e^{2x}-e^{3y}$ at (0, 0).

[6 *marks*]

9 (a) Given $f(x)=\begin{cases}\dfrac{x^3-64}{x-4}, & x\neq 4\\ 40, & x=4.\end{cases}$

(i) Find $\lim\limits_{x\to 4} f(x)$.

[4 *marks*]

(ii) Is f continuous at $x=4$? Give your reason.

[3 *marks*]

(b) Determine the values of A and B such that the function

$$h(x)=\begin{cases}Ax-B, & x\leq -1\\ 2x^2+3Ax+B, & -1<x\leq 1\\ 4, & x>1.\end{cases}$$

is continuous for all values of x.

[6 *marks*]

10 (a) Given $\tan\frac{\pi}{3}=\sqrt{3}$ and $\tan\frac{\pi}{4}=1$.

Express $\tan\frac{7\pi}{12}$ in the form of $a+\sqrt{b}$ where a and b are integers.

Hence, show that $\tan\left(\frac{7\pi}{6}\right)=\frac{1}{\sqrt{b}}$.

[6 *marks*]

(b) Find R and α such that the expression $9\sin\theta+12\cos\theta$ can be expressed in the form of $R\sin(\theta+\alpha)$, where $R>0$, $0^\circ<\alpha<90^\circ$.

Hence, if $9\sin\theta+12\cos\theta=5$, solve for θ in the interval $0^\circ\leq\theta\leq360^\circ$.

[9 *marks*]

END OF QUESTION PAPER