

QM016/1
Mathematics
Paper 1
Semester I
2008/2009
2 hours

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Matematik
Kertas 1
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BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **13** halaman bercetak.

This booklet consists of 13 printed pages.

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π , e , surd, fractions or correct to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Arithmetic Series:

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Geometric Series:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{for } r < 1$$

Binomial Expansions:

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n, \text{ where } n \in \mathbb{N} \text{ and}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \text{ for } |x| < 1$$

1. Express $\frac{5x^2 + 3x + 8}{(1-x^2)(1+x)}$ in partial fractions.

[5 marks]

2. The fifth term and the tenth term of a geometric series are 3125 and 243 respectively.

- (a) Find the value of common ratio, r of the series.

[3 marks]

- (b) Determine the smallest value of n such that $\frac{S_\infty - S_n}{S_\infty} < 0.02$, where S_n is the sum of the first n term and S_∞ is the sum to infinity of the geometric series.

[3 marks]

3. Solve the equation $3\log_x 3 + \log_3 \sqrt[3]{x} = \frac{10}{3}$.

[7 marks]

4. Determine the interval of x satisfying the inequality $|x+2| > 10-x^2$.

[7 marks]

5. The roots of the quadratic equation $2x^2 = 4x - 1$ are α and β .

- (a) Find the values of $\alpha^2 + \beta^2$ and $\alpha^3\beta + \alpha\beta^3$.

[5 marks]

- (b) Form a new quadratic equation whose roots are $(\alpha-2)$ and $(\beta-2)$.

[5 marks]

6. (a) Given $z_1 = 1 - i$ and $z_2 = 4 + 2i$. Express $\frac{z_1^2}{z_1 - z_2}$ in the form of $a + bi$, where a and b are real numbers. Hence, determine $\left| \frac{z_1^2}{z_1 - z_2} \right|$.

[5 marks]

- (b) Given that $z = x + iy$, where x and y are the real numbers and \bar{z} is the complex conjugate of z . Find the positive values of x and y so that

$$\frac{1}{\bar{z}} + \frac{2}{z} = 3 - i.$$

[6 marks]

7. (a) The r th term of an arithmetic progression is $(1 + 6r)$. Find in terms of n , the sum of the first n terms of the progression.

[4 marks]

- (b) (i) Show that $\frac{1}{\sqrt{9-x}} = \frac{1}{3} \left(1 - \frac{x}{9} \right)^{-\frac{1}{2}}$.

[3 marks]

- (ii) Find the first three terms in the binomial expansion of $\left(1 - \frac{x}{9} \right)^{-\frac{1}{2}}$ in ascending powers of x and state the range of values of x for which this expansion is valid.

[3 marks]

- (iii) Find the first three terms in the expansion of $\frac{3(1+x)}{\sqrt{9-x}}$ in ascending powers of x .

[3 marks]

8. (a) Given the matrices $P = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 2 & -3 \\ 2 & -1 & 0 \\ -3 & 0 & 3 \end{bmatrix}$. Find PQ and

hence, determine P^{-1} .

[4 marks]

- (b) The following table shows the quantities (kg) and the amount paid (RM) for the three types of items bought by three housewives in a supermarket.

Housewives	Sugar (kg)	Flour (kg)	Rice (kg)	Amount Paid (RM)
Aminah	3	6	3	16.50
Malini	6	3	6	21.30
Swee Lan	3	6	6	21.00

The prices in RM per kilogram (kg) of sugar, flour and rice are x , y and z respectively.

- (i) Form a system of linear equations from the above information and write the system of linear equations in the form of matrix equation $AX = B$.

[3 marks]

- (ii) Rewrite $AX = B$ above in the form $kPX = B$, where $A = kP$ (P is the matrix in (a)) and k is a constant. Determine the value of k and hence find the values of x , y and z .

[6 marks]

9. Polynomial $P(x) = mx^3 - 8x^2 + nx + 6$ can be divided exactly by $x^2 - 2x - 3$. Find the values of m and n . Using these values of m and n , factorize the polynomial completely. Hence, solve the equation

$$3x^4 - 14x^3 + 11x^2 + 16x - 12 = 0$$

- using the polynomial $P(x)$.

[13 marks]

10. Matrix A is given by $A = \begin{bmatrix} 0 & 1 & 1 \\ 5 & 1 & -1 \\ 2 & -3 & -3 \end{bmatrix}$.

- (a) Find
- (i) the determinant of A ,
 - (ii) the minor of A and
 - (iii) the adjoint of A .

[9 marks]

- (b) Based on part (a) above, find A^{-1} . Hence, solve the simultaneous equations

$$y + z = \frac{3}{2}$$

$$5x + y - z = 9$$

$$2x - 3y - 3z = \frac{3}{2}$$

[6 marks]

END OF BOOKLET