

QM016/2
Mathematics
Paper 2
Semester I
Session 2006/2007
2 hours

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Matematik
Kertas 2
Semester I
Sesi 2006/2007
2 jam



BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 2
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.

This booklet consists of 11 printed pages.

KANG KOOI WEI

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks allocated for each question or section is shown in the bracket at the end of each question or section.

All steps must be shown clearly.

Only non-programmable scientific calculator can be used.

Numerical answers can be given in the form of π , e , surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

Differentiation

If $y = g(t)$ and $x = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Integration

$$\int u dv = uv - \int v du$$

1. Evaluate each of the following limits, if it exists.

(a) $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$. [3 marks]

(b) $\lim_{x \rightarrow +\infty} \frac{\sqrt{3x^4+x}}{x^2-6}$. [3 marks]

2. Given that $x = \frac{1}{1+t^2}$ and $y = \frac{1-t^2}{t}$, where t is a non-zero parameter.

Show that

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{1+t^2}{t} \right)^3.$$

Hence find its value at the point $(\frac{1}{2}, 0)$. [6 marks]

3. If $y = e^{-x} \ln x$, show that

$$x^2 \left(\frac{d^2y}{dx^2} + \frac{dy}{dx} \right) + e^{-x} (1+x) = 0. \quad [6 \text{ marks}]$$

4. Let

$$g(x) = \begin{cases} x & , \quad x \leq 1 \\ ax+b & , \quad 1 < x < 4 \\ -2x & , \quad x \geq 4. \end{cases}$$

Find the values of a and b so that g is continuous on the interval $(-\infty, \infty)$.

[7 marks]

5. (a) Given $f(x) = \begin{cases} 5x^2 + m, & x < 2 \\ k, & x = 2 \\ mx^3 - 1, & x > 2. \end{cases}$

Find the value of m such that $\lim_{x \rightarrow 2} f(x)$ exists. Hence find the value of k such that f is continuous at $x = 2$. [6 marks]

(b) Given a function f on a closed interval $[-2,4]$ as follows:

$$f(x) = \begin{cases} \frac{x^2 + 2}{(x-1)(x+3)}, & -2 \leq x < 4 \\ 5, & x = 4. \end{cases}$$

Find the intervals on $[-2,4]$ where f is continuous .

[5 marks]

6. Let $x^2y^2 + 2xy + 4y = 4$.

(a) Find the values of A , B and C if $\frac{dy}{dx} = \frac{Ay(1+xy)}{x(xy+B)+C}$.

[6 marks]

(b) Determine the value of $\frac{d^2y}{dx^2}$ at the point $(2,2)$.

[6 marks]

7. A function f is defined by

$$f(x) = |x+1| - 2.$$

(a) Sketch the graph of f . Hence, determine its domain and range.

[4 marks]

(b) Is f differentiable in its domain? Justify your answer.

[4 marks]

(c) Evaluate $\int_{-4}^4 f(x) dx$.

[4 marks]

8. Let R be a region bounded by $y = \sqrt{x} \ln x$, $y = 0$, $x = 1$ and $x = 4$. Find

(a) the area of R ,

[5 marks]

(b) the volume of revolution when R is rotated through 360° about the x - axis.

[7 marks]

9. Express $\frac{2x^4 + 4x^2 + 1}{x^3 + x}$ as partial fractions.

[6 marks]

Hence, evaluate $\int_1^2 \frac{2x^4 + 4x^2 + 1}{x^3 + x} dx$.

[7 marks]

10. The functions f , g and h are defined by

$$f(x) = x^2 - 1, \quad g(x) = \sqrt{x}, \quad x \geq 0 \quad \text{and} \quad h(x) = \frac{1}{x}, \quad x \neq 0.$$

(a) Show that

$$F(x) = (h \circ g \circ f)(x) = \frac{1}{\sqrt{x^2 - 1}}. \quad [2 \text{ marks}]$$

(b) State the domain and range of F . [2 marks]

(c) Find the vertical and horizontal asymptotes of F . [2 marks]

(d) Sketch the graph of F . Determine its points of discontinuity and hence state the largest interval where F is continuous. [4 marks]

(e) For $x > 1$, find $F^{-1}(x)$ and hence determine real p such that

$$F^{-1}(p) = \sqrt{\frac{3}{2}} F(p). \quad [5 \text{ marks}]$$

END OF QUESTION BOOKLET