QM016/1
Mathematics
Paper 1
Semester I
Session 2005/2006
2 hours

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Matematik
Kertas 1
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BAHAGIAN MATRIKULASI
KEMENTERIAN PELAJARAN MALAYSIA
MATRICULATION DIVISION
MINISTRY OF EDUCATION MALAYSIA

PEPERIKSAAN SEMESTER PROGRAM MATRIKULASI
MATRICULATION PROGRAMME EXAMINATION

MATEMATIK
Kertas 1
2 jam

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU.
DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Kertas soalan ini mengandungi **11** halaman bercetak.
This booklet consists of **11** *printed pages.*

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INSTRUCTIONS TO CANDIDATE:

This question booklet consists of **10** questions.

Answer **all** questions.

The full marks for each question or section are shown in the bracket at the end of each of the question or section.

All steps must be shown clearly.

Only non-programmable scientific calculators can be used.

Numerical answers can be given in the form of π, e, surd, fractions or up to three significant figures, where appropriate, unless stated otherwise in the question.

LIST OF MATHEMATICAL FORMULAE

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

For a geometric series:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

dengan $n \in N$ dan $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

1. Find the values of x satisfying the equation

$$\log_4\left(x^4+4\right)=1+\log_4\left(x^2+4\right).$$ [5 marks]

2. Polynomial $P(x)=2x^3+ax^2-x+b$ has $(x+1)$ as a factor and leaves a remainder 12 when divided by $(x-3)$. Determine the values of a and b.

[6 marks]

3. The quadratic equation $x^2+2(k+1)x+(2k+5)=0$ has roots α and β. If the roots are equal, show that $k=2$. Hence, find the quadratic equation with roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

[7 marks]

4. The third and the sixth terms of a geometric series are $\frac{1}{2}$ and $\frac{1}{16}$. Determine the values of the first term and the common ratio. Hence, find the sum of the first nine terms of the series.

[7 marks]

5. Solve the following inequalities:

(a) $7x^2+x-6\leq x^2-4$. [4 marks]

(b) $\left|\frac{x+3}{x-1}\right|>3$. [6 marks]

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6. Expand $\dfrac{1}{(3-x)^3}$ up to the term x^3 and determine the interval of x for which the expansion is valid. Hence, approximate $\dfrac{1}{(2.9)^3}$ correct to four decimal places.

[12 marks]

7. Given $A=\begin{bmatrix}-1 & 0 & -2\\ 2 & 1 & 0\\ -1 & 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & -1\\ -1 & 0\\ 2 & -1\end{bmatrix}$ and $C=\begin{bmatrix}2 & -1 & 0\\ 3 & 0 & -1\end{bmatrix}$.

(a) Find matrix $D = A-(BC)^T$. [5 marks]

(b) Show that $|AD| = |DA|$. [7 marks]

8. Two factors of the polynomial $P(x) = x^3 + ax^2 + bx - 6$ are $(x+1)$ and $(x-2)$. Determine the values of a and b, and find the third factor of the polynomial. Hence, express

$$\frac{2x^2-5x-13}{P(x)}$$

as a sum of partial fractions. [13 marks]

9. (a) Given two complex numbers $z_1 = 1+3i$ and $z_2 = 2-i$, express $\dfrac{z_1+z_2}{z_1 z_2}$ in the form of $a+bi$, where a and b are real numbers. Hence, determine $\left|\dfrac{z_1+z_2}{z_1 z_2}\right|$. [6 marks]

(b) Solve the equation $5(2^{x+1}) - 4^x = 16$. [7 marks]

10. Consider the system of linear equations

$$
\begin{aligned}
x_1 - 2x_2 + 3x_3 - 1 &= 0 \\
x_1 + mx_2 + 2x_3 &= 2 \\
-2x_1 + m^2x_2 - 4x_3 + 4 &= 3m
\end{aligned}
$$

where m is a constant.

(a) Write the above system of linear equations in augmented matrix, $[A \mid B]$.

[2 marks]

(b) By using row operations, show that the above augmented matrix can be reduced to

$$
\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & m+2 & -1 & 1 \\ 0 & 0 & m & 2m \end{array}\right].
$$

[6 marks]

(i) Solve the above system of linear equations for $m = 1$.

[5 marks]

(ii) State the condition of m for which the system of linear equations has an infinite number of solutions and has no solution.

[2 marks]

END OF QUESTION BOOKLET