KEMENTERIAN PENDIDIKAN MALAYSIA

QS 025/1

Matriculation Programme Examination

Semester I

Session 2016/2017

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1. Find the angle between the line $l: \langle x, y, z\rangle = \langle 1,3,-1\rangle + t\langle 2,1,0\rangle$ and the plane $\Pi: 3x - 2y + z = 5$.

SOLUTION

$\underline{v} = \langle 2, 1, 0\rangle$

$\underline{n} = \langle 3, -2, 1\rangle$

$\underline{v}.\underline{n} = \langle 2, 1, 0\rangle.\langle 3, -2, 1\rangle$

$= (2)(3) + (1)(-2) + (0)(1)$

$= 6 - 2 + 0$

$= 4$

$|\underline{v}| = \sqrt{2^2 + 1^2 + 0^2}$

$= \sqrt{5}$

$|\underline{n}| = \sqrt{3^2 + (-2)^2 + 1^2}$

$= \sqrt{14}$

$$\cos\theta = \frac{\underline{v}.\underline{n}}{|\underline{v}|.|\underline{n}|}$$

$$= \frac{4}{\sqrt{5}.\sqrt{14}}$$

$= 0.4781$

$\theta = cos^{-1}0.4781$

$= 61.4°$ (Angle between the line l and the normal vector of plane Π)

Angle between the line l and the vector of plane $\Pi = 90° - 61.4° = 28.6°$

2. Solve $\int \frac{e^{2x}}{1-e^{2x}}\,dx$.

SOLUTION

$$\int \frac{e^{2x}}{1-e^{2x}}\,dx$$

$$u = 1 - e^{2x}$$

$$\frac{du}{dx} = -2e^{2x}$$

$$du = -2e^{2x}dx$$

$$e^{2x}dx = -\frac{du}{2}$$

$$\int \frac{e^{2x}}{1-e^{2x}}\,dx = -\frac{1}{2}\int \frac{1}{u}\,du$$

$$= -\frac{1}{2}\ln|u| + c$$

$$= -\frac{1}{2}\ln|1-e^{2x}| + c$$

Alternative

$$\int \frac{e^{2x}}{1-e^{2x}}\,dx$$

$$u = e^{2x}$$

$$\frac{du}{dx} = 2e^{2x}$$

$$du = 2e^{2x}dx$$

$$e^{2x}dx = -\frac{1}{2}du$$

$$\int \frac{e^{2x}}{1-e^{2x}}\,dx = \int \frac{1}{1-u}\left(\frac{1}{2}du\right)$$

$$= \frac{1}{2}\int \frac{1}{1-u}\,du$$

$$= -\frac{1}{2}\ln|1-u| + c$$

$$= -\frac{1}{2}\ln|1-e^{2x}| + c$$

3. Given four points $A = (-2, -8, 4), B(2, -\omega, -1), C = (0, -9, 0)\ and\ D = (-4, -3, 7)$.
Determine the value of ω if $\overrightarrow{AB}.(\overrightarrow{AC} \text{ x } \overrightarrow{AD}) = 64$.

SOLUTION

$A = (-2, -8, 4),\ \ B(2, -\omega, -1), C = (0, -9, 0)\ and\ D = (-4, -3, 7)$

$\overrightarrow{\boldsymbol{AB}} = \overrightarrow{\boldsymbol{OB}} - \overrightarrow{\boldsymbol{OA}}$

$= \langle 2, -\omega, -1\rangle - \langle -2, -8, 4\rangle$

$= \langle 4, 8 - \omega, -5\rangle$

$\overrightarrow{\boldsymbol{AC}} = \overrightarrow{\boldsymbol{OC}} - \overrightarrow{\boldsymbol{OA}}$

$= \langle 0, -9, 0\rangle - \langle -2, -8, 4\rangle$

$= \langle 2, -1, -4\rangle$

$\overrightarrow{\boldsymbol{AD}} = \overrightarrow{\boldsymbol{OD}} - \overrightarrow{\boldsymbol{OA}}$

$= \langle -4, -3, 7\rangle - \langle -2, -8, 4\rangle$

$= \langle -2,5, 3\rangle$

$$\overrightarrow{\boldsymbol{AC}}\boldsymbol{X}\ \overrightarrow{\boldsymbol{AD}} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & -1 & -4 \\ -2 & 5 & 3 \end{vmatrix}$$

$= 17\boldsymbol{i} + 2\boldsymbol{j} + 8\boldsymbol{k}$

$= \langle 17,2,8\rangle$

$\overrightarrow{AB}.(\overrightarrow{AC} \text{ x } \overrightarrow{AD}) = 64$

$\langle 4, 8 - \omega, -5\rangle.\langle 17,2,8\rangle = 64$

$(4)(17) + (8 - \omega)(2) + (-5)(8) = 64$

$68 + 16 - 2\omega - 40 = 64$

$44 - 2\omega = 64$

$2\omega = -20$

$\omega = -10$

4. Find the vertex, focus and directrix for the parabola $y^2 + 64 = 8y - 16x$. Hence, sketch and label the vertex, focus and directrix for the curve.

SOLUTION

$y^2 + 64 = 8y - 16x$

$y^2 - 8y = -16x - 64$

$y^2 - 8y + \left(\frac{-8}{2}\right)^2 = -16x - 64 + \left(\frac{-8}{2}\right)^2$

$(y-4)^2 = -16x - 64 + 16$

$(y-4)^2 = -16x - 48$

$(y-4)^2 = -16(x+3)$

Compare
$(y-4)^2 = -16(x+3)$
$(y-k)^2 = 4p(x-h)$

$h = -3, \quad k = 4, \quad 4p = -16 ==> p = -4$

$\boldsymbol{Vertex}$

$V(h,k) = V(-3,4)$

$\boldsymbol{Focus},$

$F(h+p,k) = F(-3-4,4)$

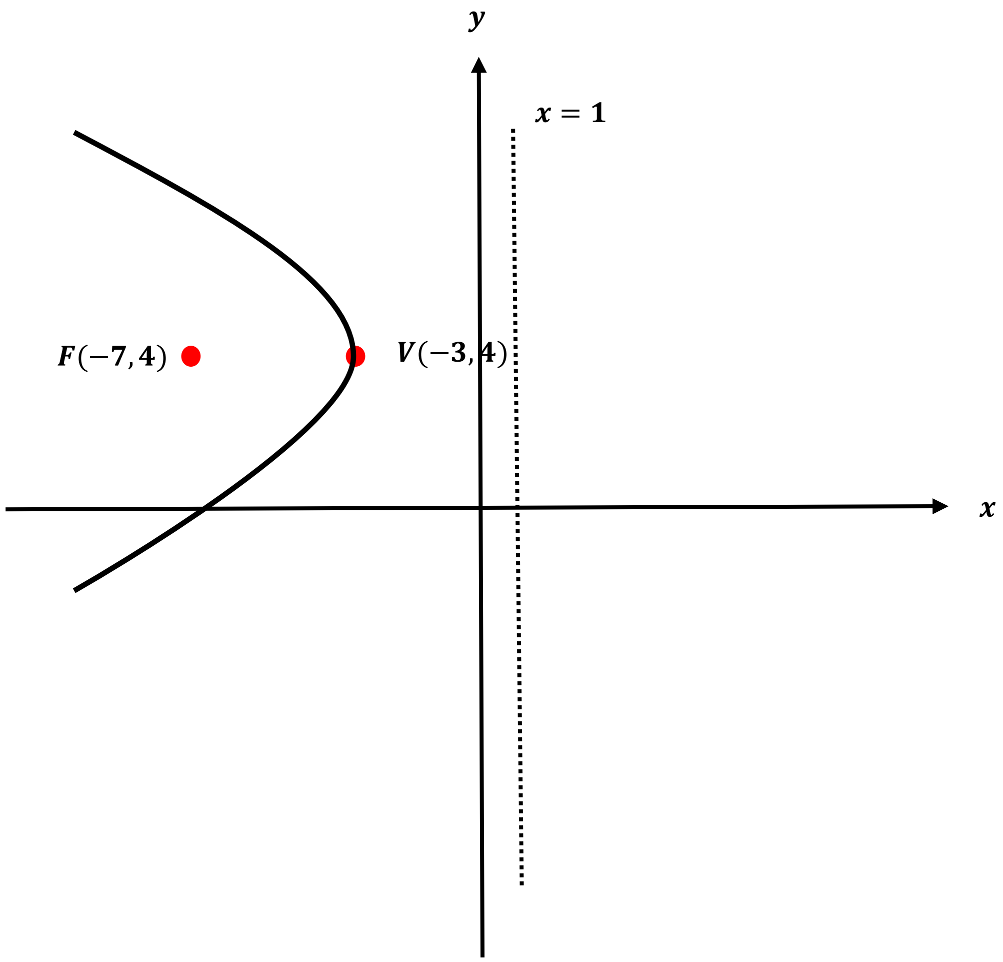
$= F(-7,4)$

$\boldsymbol{Directrix},$

$x = h - p$

$= -3 + 4$

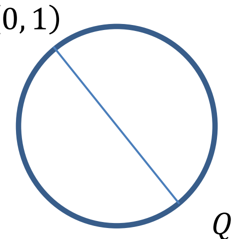
$x = 1$


y
x = 1
F(−7, 4)
V(−3, 4)
x

5. The end points of the diameter of a circle are $P(0,1) and\ Q(3,-3)$.
   a) Determine an equation of the circle.
   b) Find an equation of the tangent line to the circle at the point $P(0,1)$.

SOLUTION

a) Diameter: $P(0,1) and\ Q(3,-3)$



$$Center, C(h,k) = \left(\frac{\mathbf{0+3}}{\mathbf{2}}, \frac{\mathbf{1-3}}{\mathbf{2}}\right)$$

$$= \left(\frac{3}{2}, -1\right)$$

$$Diameter = \sqrt{(3-0)^2 + (-3-1)^2}$$

$$= \sqrt{25}$$

$$= 5$$

$$Radius, r = \frac{1}{2}(diameter)$$

$$= \frac{5}{2}$$

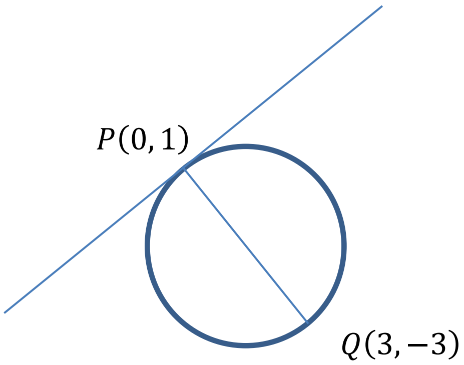
Standard equation of circle

$$(\boldsymbol{x} - \boldsymbol{h})^2 + (\boldsymbol{y} - \boldsymbol{k})^2 = \boldsymbol{r}^2$$

$$\left(x - \frac{3}{2}\right)^{\mathbf{2}} + (y+1)^{\mathbf{2}} = \left(\frac{5}{2}\right)^{\mathbf{2}}$$

$$\left(x-\frac{3}{2}\right)^{2}+(y+1)^{2}=\frac{25}{4}$$

b)



Slope of PQ

$$m_{pq}=\frac{-3-1}{3-0}$$

$$=-\frac{4}{3}$$

Slope of tangent line

$$m_t=-\frac{1}{m_{pq}}$$

$$=-\frac{1}{-\frac{4}{3}}$$

$$=\frac{3}{4}$$

Equation of the tangent line to the circle at the point $P(0,1)$

$$y-y_1=m_t(x-x_1)$$

$$y - 1 = \frac{3}{4}(x - 0)$$

$$y = \frac{3}{4}x + 1$$

6. In a Chemistry experiment, sodium hydroxide, NaOH, reacts with hydrochloric acid, HCl, to form sodium chloride salt, NaCl, and water. Before the reaction starts, no NaCl salt is formed. At time t (minute), the mass of NaCl salt formed is x grams and the rate of change of x is given by $\frac{dx}{dt} = \alpha(50 - x)$, where α is a positive constant.
   a) Find the general solution for the above equation.
   b) Find the particular solution if 35 grams of NaCl salt has formed in the first 30 minutes.
   c) Hence, find
      i. The mass of NaCl salt formed in 60 minutes.
      ii. The time taken to form 40 grams of NaCl salt.

SOLUTION

a) $\frac{dx}{dt} = \alpha(50 - x)$

$$\frac{1}{(50-x)}dx = \alpha\, dt$$

$$\int \frac{1}{(50-x)}dx = \int \alpha\, dt$$

$$-\ln(50 - x) = \alpha t + c$$

$$\ln(50 - x) = -(\alpha t + c)$$

$$50 - x = e^{-(\alpha t + c)}$$

$$50 - x = e^{-\alpha t}.e^{-c}$$

$$50 - x = Ae^{-\alpha t}; \qquad A = e^{-c}$$

$\boldsymbol{When\ t = 0}, \qquad \boldsymbol{x = 0}$

$$50 - 0 = Ae^{-0}$$

$$A = 50$$

General Solution

$$50 - x = 50e^{-\alpha t}$$

$$50 - 50e^{-\alpha t} = x$$

$$x = 50(1 - e^{-\alpha t})$$

b) $When\ t = 30, \quad x = 35$

$$35 = 50(1 - e^{-30\alpha})$$

$$1 - e^{-30\alpha} = \frac{35}{50}$$

$$1 - e^{-30\alpha} = 0.7$$

$$e^{-30\alpha} = 1 - 0.7$$

$$e^{-30\alpha} = 0.3$$

$$\ln e^{-30\alpha} = \ln 0.3$$

$$-30\alpha = -1.2040$$

$$\alpha = \frac{-1.2040}{-30}$$

$$\alpha = 0.0401$$

Particular

$$x = 50(1 - e^{-0.0401t})$$

d) i) $When\ t = 60$

$$x = 50\left[1 - e^{-0.0401(60)}\right]$$

$$= 50\left[1 - e^{-0.0401(60)}\right]$$

$$= 45.5$$

$ii)\ When\ x = 40$

$$40 = 50(1 - e^{-0.0401t})$$

$$\frac{40}{50} = 1 - e^{-0.0401t}$$

$$0.8 = 1 - e^{-0.0401t}$$

$$e^{-0.0401t} = 1 - 0.8$$

$$e^{-0.0401t} = 0.2$$

$$\ln e^{-0.0401t} = \ln 0.2$$

$$-0.0401t = \ln 0.2$$

$$t = \frac{\ln 0.2}{-0.0401}$$

$$t = 40.1$$

7. (a) Show that equation $-4x^2 + 5x + 7 = 0$ has a root on the interval $[-2, 0]$. Use the Newton-Raphson method to find the root of the equation correct to four decimal places.

(b) Estimate the value of $\int_{-\pi}^{0} x \cos x \, dx$ using trapezoidal rule with subinterval $\frac{\pi}{4}$. Give your answer correct to four decimal places.

SOLUTION

a) $-4x^2 + 5x + 7 = 0$

$f(x) = -4x^2 + 5x + 7$

When x = -2

$f(-2) = -4(-2)^2 + 5(-2) + 7$

$= -16 - 10 + 7$

$= -19 < 0$

When x = 0

$f(0) = -4(0)^2 + 5(0) + 7$

$= 7 > 0$

$Since \ f(-2) < 0 \ and \ f(0) > 0, therefore$

$equation \ -4x\text{^}2 + 5x + 7 = 0 \ has \ a \ root \ on \ the \ interval \ [-2, 0]$

Newton-Raphson Method

$$f(x) = -4x^2 + 5x + 7$$

$$f'(x) = -8x + 5$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x - \frac{-4x^2 + 5x + 7}{-8x + 5}$$

$$x_{n+1} = x - \frac{4x^2 - 5x - 7}{8x - 5}$$

$$x_0 = -1$$

$$x_1 = -1 - \frac{4(-1)^2 - 5(-1) - 7}{8(-1) - 5} = -0.8462$$

$$x_2 = -0.8462 - \frac{4(-0.8462)^2 - 5(-0.8462) - 7}{8(-0.8462) - 5} = -0.8381$$

$$x_3 = -0.8381 - \frac{4(-0.8381)^2 - 5(-0.8381) - 7}{8(-0.8381) - 5} = -0.8381$$

b) $\int_{-\pi}^{0} x \cos x \, dx$

$$h = \frac{\pi}{4}$$

x	$f(x) = x \cos x$	
	First and last ordinate	**Remaining ordinat**
$-\pi$	3.14159	
$-\frac{3\pi}{4}$		1.66608
$-\frac{2\pi}{4}$		0
$-\frac{\pi}{4}$		-0.55536
0	0	
Total	**3.14159**	**1.11072**

$$\int_{a}^{b} f(x)\ dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})]$$

$$\int_{-\pi}^{0} x \cos x \ dx = \frac{\pi}{2(4)}[(3.14159) + 2(1.11072)]$$

$$= \frac{\pi}{2(4)}[(3.14159) + 2(1.11072)]$$

$$= 2.1061$$

8. Given the curve $y = 4x^2$ and the line $y = 6x$.

a) Find the intersection points.

b) Sketch the region enclosed by the curve and the line.

c) Calculate the area of the region enclosed by the curve and the line.

d) Calculate the volume of the solid generated when the region is revolved completely about the y-axis.

SOLUTION

a) $y = 4x^2$ .... (1)

$y = 6x$ .... (2)

$Substitute\ (2) into\ (1)$

$6x = 4x^2$

$4x^2 - 6x = 0$

$2x(2x - 3) = 0$

$2x = 0 \qquad 2x - 3 = 0$

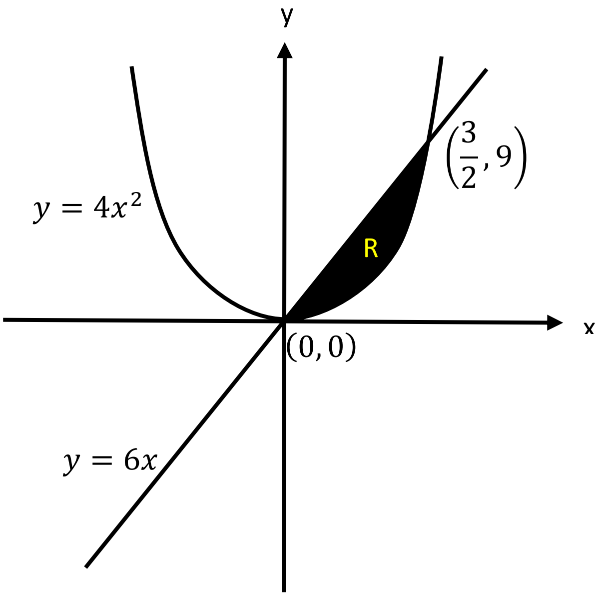
$x = 0 \qquad x = \frac{3}{2}$

$When\ x = 0$ ➔ $y = 6(0) = 0$ ➔ $(0, 0)$

$When\ x = \frac{3}{2}$ ➔ $y = 6\left(\frac{3}{2}\right) = 9$ ➔ $\left(\frac{3}{2}, 9\right)$

$\therefore Intersection\ points: (0, 0)\ and\ \left(\frac{3}{2}, 9\right)$

b)



c) $Area, R = \int_0^{\frac{3}{2}} 6x - 4x^2 \, dx$

$$= \left[\frac{6x^2}{2} - \frac{4x^3}{3}\right]_0^{\frac{3}{2}}$$

$$= \left[3x^2 - \frac{4x^3}{3}\right]_0^{\frac{3}{2}}$$

$$= \left[3\left(\frac{3}{2}\right)^2 - \frac{4\left(\frac{3}{2}\right)^3}{3}\right] - \left[3(0)^2 - \frac{4(0)^3}{3}\right]$$

$$= \left[3\left(\frac{9}{4}\right) - \frac{4\left(\frac{27}{8}\right)}{3}\right] - [0]$$

$$= \left[\frac{27}{4} - \frac{27}{6}\right]$$

$$= \frac{9}{4} \; unit^2$$

d) $y = 4x^2 \Rightarrow x^2 = \frac{y}{4}$

$y = 6x \Rightarrow x = \frac{y}{6} \Rightarrow x^2 = \frac{y^2}{36}$

$$Volume, V = \pi \int_0^9 \frac{y}{4} - \frac{y^2}{36}\, dy$$

$$= \pi \left[\frac{y^2}{4(2)} - \frac{y^3}{36(3)}\right]_0^9$$

$$= \pi \left[\frac{y^2}{8} - \frac{y^3}{108}\right]_0^9$$

$$= \pi \left[\left(\frac{9^2}{8} - \frac{9^3}{108}\right) - \left(\frac{0^2}{8} - \frac{0^3}{108}\right)\right]$$

$$= \pi \left[\left(\frac{81}{8} - \frac{729}{108}\right) - 0\right]$$

$$= \pi \left(\frac{27}{8}\right)$$

$$= \frac{27}{8}\pi \ unit^3$$

9. (a) If the line $l_1: \langle x, y, z \rangle = \langle 1,1,2 \rangle + t\langle 2,-1,3 \rangle$ does not intersect with the plane $\Pi_1: Ax + By + Cz = 0$, show that $2A - B + 3C = 0$. Hence, find the equation of plane Π_1 if the plane passes through the point $(1, 0, 1)$.

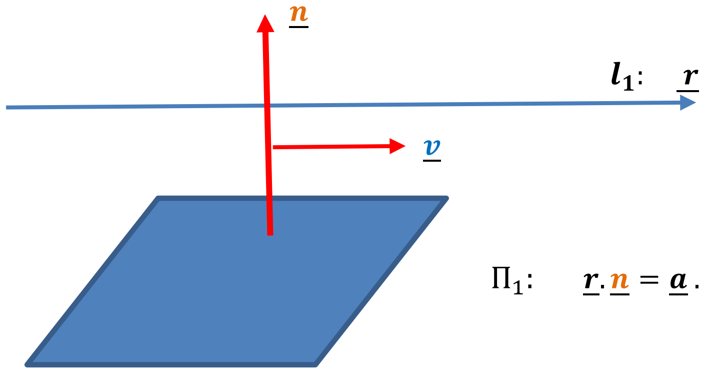
(b) Given the line $l_2: x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3$, the plane $\Pi_2: x - y + 2z = 0$, and a point $(x_0, y_0, z_0) \neq (0, 0, 0)$ is on the plane.

(i) If l_2 is perpendicular to the plane Π_2, show that $\langle v_1, v_2, v_3 \rangle = v_2\langle -1, 1, -2 \rangle;\ v_2 \neq 0$.

(ii) Give one example of the equation of straight line which satisfy part 9(b)(i)

SOLUTION

a) **A line does not intersect with a plane ➔ line is parallel to the plane ➔ $\underline{\boldsymbol{n}}$ is perpendicular to $\underline{\boldsymbol{v}}$ ➔ $\underline{\boldsymbol{n}} \cdot \underline{\boldsymbol{v}} = \mathbf{0}$ (See the diagram below)**



l_1: $\underline{\boldsymbol{r}} = \underline{\boldsymbol{a}} + t\underline{\boldsymbol{v}}$

Π_1: $\underline{\boldsymbol{r}} \cdot \underline{\boldsymbol{n}} = \underline{\boldsymbol{a}} \cdot \underline{\boldsymbol{n}}$

$l_1: \langle x, y, z \rangle = \langle 1,1,2 \rangle + t\langle 2,-1,3 \rangle$ ➔ $\underline{v} = \langle 2,-1,3 \rangle$

$\Pi_1: Ax + By + Cz = 0,$ ➔ $\underline{n} = \langle A, B, C \rangle$

$\underline{\boldsymbol{n}} \cdot \underline{\boldsymbol{v}} = \mathbf{0}$

$\langle A, B, C \rangle \cdot \langle 2,-1,3 \rangle = \mathbf{0}$

$2A - B + 3C = 0$

$\Pi_1: Ax + By + Cz = 0$

$2A - B + 3C = 0 \qquad \ldots\ldots\ldots\ldots\ldots(1)$

Point $(1, 0, 1)$ is on the plane $Ax + By + Cz = 0$

$A(1) + B(0) + C(1) = 0$

$A + C = 0$

$C = -A \qquad \ldots\ldots\ldots\ldots\ldots(2)$

$Substitute\ (2)\ into\ (1)$

$2A - B + 3(-A) = 0$

$2A - B - 3A = 0$

$-B - A = 0$

$B = -A \qquad \ldots\ldots\ldots\ldots\ldots(3)$

$\therefore\ Ax + By + Cz = 0$ ➔ $Ax + (-A)y + (-A)z = 0$

$Ax - Ay - Az = 0$

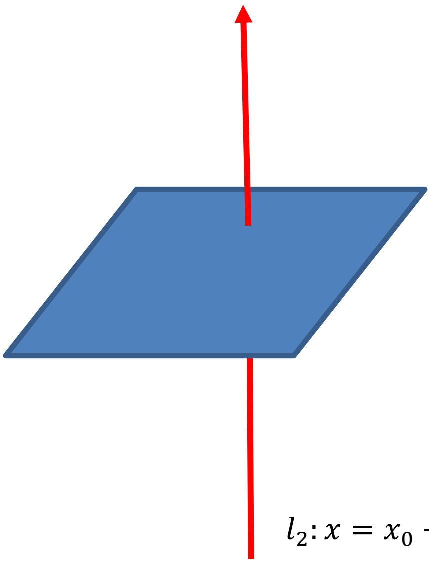
$A(x - y - z) = 0$

$x - y - z = 0$

b) line $l_2 : x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3$

$\Pi_2 : x - y + 2z = 0$

i)



l_2 *is perpendicular to the plane* Π_2 ➔ $\underline{v}$ *is parallel to* $\underline{n}$ ➔ $\underline{v} \; x \; \underline{n} = 0$

$\underline{v} = \langle v_1, v_2, v_3 \rangle$

$\underline{n} = \langle 1, -1, 2 \rangle$

$\underline{v} \; x \; \underline{n} = 0$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ v_1 & v_2 & v_3 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$[2v_2 - (-v_3)]\underline{i} - (2v_1 - v_3)\underline{j} + (-v_1 - v_2)\underline{k} = 0$

$[2v_2 + v_3]\underline{i} - (2v_1 - v_3)\underline{j} + (-v_1 - v_2)\underline{k} = 0$

$2v_2 + v_3 = 0$ ➔ $v_3 = -2v_2$

$2v_1 - v_3 = 0$

$v_1 + v_2 = 0 \quad \rightarrow \quad v_1 = -v_2$

$\therefore \langle v_1, v_2, v_3 \rangle = \langle -v_2, v_2, -2v_2 \rangle$

$= v_2 \langle -1, 1, -2 \rangle; \quad v_2 \neq 0$

ii) $\underline{v} = \underline{n} = \langle 1, -1, 2 \rangle$

At the point $(x_0, y_0, z_0) = (-1, 1, 1)$

Equation of straight line $\boldsymbol{l_2}$:

$x = -1 + t(1) \quad \rightarrow \quad x = -1 + t$

$y = 1 + t(-1) \quad \rightarrow \quad y = 1 - t$

$z = 1 + t(2) \quad \rightarrow \quad z = 1 + 2t$

10. (a) Show that the expression $\frac{4x^4+2x^2-1}{(2x-3)^2(x+1)}$ can be written as

$$x + 2 + \frac{A}{2x-3} + \frac{B}{(2x-3)^2} + \frac{C}{x+1}.$$

(b) From part 10(a), determine the values of A, B and C. Hence, solve

$$\int \frac{4x^4+2x^2-1}{(2x-3)^2(x+1)}\, dx$$

SOLUTION

a) Improper Fraction ➔ Use long division

$$\frac{4x^4+2x^2-1}{(2x-3)^2(x+1)} = \frac{4x^4+2x^2-1}{(4x^2-12x+9)(x+1)}$$

$$= \frac{4x^4+2x^2-1}{4x^3+4x^2-12x^2-12x+9x+9}$$

$$= \frac{4x^4+2x^2-1}{4x^3-8x^2-3x+9}$$

$$\begin{array}{r|l} & x+2 \\ 4x^3-8x^2-3x+9 & 4x^4+0x^3+2x^2+0x-1 \\ & \underline{4x^4-8x^3-3x^2+9x+0} \\ & 8x^3+5x^2-9x-1 \\ & \underline{8x^3-16x^2-6x+18} \\ & 21x^2-3x-19 \end{array}$$

$$\frac{4x^4+2x^2-1}{(2x-3)^2(x+1)} = x + 2 + \frac{21x^2-3x-19}{(2x-3)^2(x+1)}$$

$$= x + 2 + \frac{A}{2x-3} + \frac{B}{(2x-3)^2} + \frac{C}{(x+1)}$$

b)

$$\frac{21x^2 - 3x - 19}{(2x-3)^2(x+1)} = \frac{A}{2x-3} + \frac{B}{(2x-3)^2} + \frac{C}{(x+1)}$$

$$= \frac{A(2x-3)(x+1) + B(x+1) + C(2x-3)^2}{(2x-3)^2(x+1)}$$

$$21x^2 - 3x - 19 = A(2x-3)(x+1) + B(x+1) + C(2x-3)^2$$

$\boldsymbol{When\ x = -1}$

$$21(-1)^2 - 3(-1) - 19 = C[2(-1) - 3]^2$$

$$5 = 25C$$

$$C = \frac{5}{25}$$

$$C = \frac{1}{5}$$

$\boldsymbol{When\ x = \frac{3}{2}}$

$$21\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 19 = B\left[\left(\frac{3}{2}\right) + 1\right]$$

$$21\left(\frac{9}{4}\right) - \frac{9}{2} - 19 = B\left[\frac{5}{2}\right]$$

$$21\left(\frac{9}{4}\right) - \frac{9}{2} - 19 = \frac{5}{2}B$$

$$\frac{95}{4} = \frac{5}{2}B$$

$$B = \frac{95}{4} x \frac{2}{5}$$

$$B = \frac{19}{2}$$

$$\boldsymbol{When\ x = 0}$$

$$-19 = A[2(0) - 3][(0) + 1] + B(0 + 1) + C[2(0) - 3]^2$$

$$-19 = -3A + B + 9C$$

$$-19 = -3A + \left(\frac{19}{2}\right) + 9\left(\frac{1}{5}\right)$$

$$-19 = -3A + \left(\frac{19}{2}\right) + \left(\frac{9}{5}\right)$$

$$-19 = -3A + \frac{113}{10}$$

$$3A = \frac{113}{10} + 19$$

$$3A = \frac{303}{10}$$

$$A = \frac{101}{10}$$

$$\therefore A = \frac{101}{10}; B = \frac{19}{2}; \quad C = \frac{1}{5}$$

$$\frac{4x^4 + 2x^2 - 1}{(2x-3)^2(x+1)} = x + 2 + \frac{101}{10(2x-3)} + \frac{19}{2(2x-3)^2} + \frac{1}{5(x+1)}$$

$$\int \frac{4x^4 + 2x^2 - 1}{(2x-3)^2(x+1)}\, dx = \int x + 2 + \frac{101}{10(2x-3)} + \frac{19}{2(2x-3)^2} + \frac{1}{5(x+1)}\, dx$$

$$= \int (x+2)\; dx + \frac{101}{10}\int \frac{1}{(2x-3)} dx + \frac{19}{2}\int (2x-3)^{-2}\, dx + \frac{1}{5}\int \frac{1}{(x+1)}\, dx$$

$$= \frac{x^2}{2} + 2x + \frac{101}{(10)2}\int \frac{2}{(2x-3)} dx + \frac{19}{2}\left[\frac{(2x-3)^{-1}}{(-1)(2)}\right] + \frac{1}{5}\ln|x+1| + C$$

$$= \frac{x^2}{2} + 2x + \frac{101}{20}\ln|2x-3|\, dx - \frac{19}{4}\left[\frac{1}{(2x-3)}\right] + \frac{1}{5}\ln\lceil x+1\rceil + C$$

$$= \frac{x^2}{2} + 2x + \frac{101}{20}\ln|2x-3|\, dx - \frac{19}{4(2x-3)} + \frac{1}{5}\ln\lceil x+1\rceil + C$$