



SM015/2
MATEMATIK

2018/2019

Matriculation Programme Examination

a) Given $z_1 = 2 + 3i$ and $z_2 = 4 - 4i$. Express $\frac{(z_2)}{(z_1)} + \left[\left(\frac{i^3}{-z_2} \right) \right]$ in Cartesian form.

b) Solve

a. $\left(\frac{27}{125} \right)^2 \times \left(\frac{25}{9} \right)^{4x} = \left(\frac{9}{25} \right)^{x-3} \times \left(\frac{625}{81} \right)^2$

b. $\frac{1}{4-2x} \geq \frac{8}{x}$

c) a) The first three terms of a geometric series are $\left(3c - \frac{7}{2} \right)$, $(3c - 2)$ and 6.

Determine the value of c . Hence, find the seventh term of this series.

b) Expand $\left(\frac{3}{2}x^2 - 1 \right)^3$

d) a) Given the matrix $\begin{bmatrix} 1 & 3 & 4 \\ a+2b & 3 & 2 \\ 4 & a+b & 9 \end{bmatrix}$ such that $M_{11} = 7$ and $C_{12} = -1$,

calculate the values of a and b .

b) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 13 & -6 \\ -1 & -7 & 2 \\ -2 & 2 & 0 \end{bmatrix}$ and $C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

i. Find determinant of A by expanding first column.

ii. Evaluate $(A^2 - B^T)C$.

e) a) Given $f(x) = \left(\frac{5x+1}{4x} \right)$ and $g(x) = \frac{\sqrt{x+1}-2}{x^2-4}$. Find

i. the domain of $g(x)$.

ii. $h(x)$, if $(f \circ h)(x) = x$

b) Given $p(x) = \ln(3x + 6)$ and $q(x) = \frac{e^x}{3} - 2$. Show that $p(x)$ and $q(x)$ are

inverses of each other.

f) The polynomial $P(x) = x^4 + ax^3 - 7x^2 - 4ax + b$ has a factor $(x + 3)$ and remainder 60 when divided by $(x - 3)$. Find the values of a and b . Hence, factorise $P(x)$ completely.

g) a) Express $12 \cos \theta + 7 \sin \theta$ in the form of $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$

b) Hence, show that the maximum value of $\frac{1}{12 \cos \theta + 7 \sin \theta + 15}$ is $\frac{1}{32}(15 + \sqrt{193})$.

h) The function $g(x)$ is defined by

$$g(x) = \begin{cases} 2 & , x \leq 2 \\ \frac{x-2}{\sqrt{2x}-2} & , 2 < x \leq 8 \\ \frac{|8-x|}{x-8} & , x > 8 \end{cases}$$

Find

a) $\lim_{x \rightarrow 2^+} g(x)$

b) $\lim_{x \rightarrow 8^+} g(x)$

i) a) Find the derivative of $f(x) = \frac{6}{\sqrt{x}}$ using the first principle.

b) Find the value of $\frac{dy}{dx}$ when $x = 0$ for each of the following:

1. $y = \ln(9 - 2x)$

2. $y = \frac{e^{-3x}}{\sqrt{3x+1}}$

j) Given $f(x) = \frac{3x}{x^2+9}$, where $x > 0$. Find the coordinates of the stationary point and state its nature.

1. Given $z_1 = 2 + 3i$ and $z_2 = 4 - 4i$. Express $\frac{z_2}{z_1} + \left[\left(\frac{i^3}{-z_2} \right) \right]$ in Cartesian form.

SOLUTION

$$z_1 = 2 + 3i$$

$$z_2 = 4 - 4i$$

$$\begin{aligned} \frac{z_2}{z_1} + \left[\left(\frac{i^3}{-z_2} \right) \right] &= \frac{4 - 4i}{2 - 3i} + \left[\left(\frac{i^3}{-(4 - 4i)} \right) \right] \\ &= \frac{4 - 4i}{2 - 3i} - \frac{i^3}{4 - 4i} \\ &= \frac{(4 - 4i)(4 - 4i) - i^3(2 - 3i)}{(2 - 3i)(4 - 4i)} \\ &= \frac{16 - 16i - 16i + 16i^2 - 2i^3 + 3i^4}{8 - 8i - 12i + 12i^2} \\ &= \frac{16 - 16i - 16i + 16(-1) - 2(-i) + 3(1)}{8 - 20i + 12(-1)} \\ &= \frac{16 - 16i - 16i - 16 + 2i + 3}{8 - 20i - 12} \\ &= \frac{3 - 30i}{-4 - 20i} \\ &= \frac{(3 - 30i)}{(-4 - 20i)} \cdot \frac{(-4 + 20i)}{(-4 + 20i)} \\ &= \frac{-12 + 60i + 120i - 600i^2}{16 - 80i + 80i - 400i^2} \\ &= \frac{-12 + 60i + 120i - 600(-1)}{16 - 80i + 80i - 400(-1)} \\ &= \frac{588 + 180i}{416} \end{aligned}$$

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$= \frac{588}{416} + \frac{180}{416}i$$
$$= \frac{147}{104} + \frac{45}{104}i$$

2. Solve

a. $\left(\frac{27}{125}\right)^2 \times \left(\frac{25}{9}\right)^{4x} = \left(\frac{9}{25}\right)^{x-3} \times \left(\frac{625}{81}\right)^2$

b. $\frac{1}{4-2x} \geq \frac{8}{x}$

SOLUTION

a. $\left(\frac{27}{125}\right)^2 \times \left(\frac{25}{9}\right)^{4x} = \left(\frac{9}{25}\right)^{x-3} \times \left(\frac{625}{81}\right)^2$

$$\frac{\left(\frac{25}{9}\right)^{4x}}{\left(\frac{9}{25}\right)^{x-3}} = \frac{\left(\frac{625}{81}\right)^2}{\left(\frac{27}{125}\right)^2}$$

$$\frac{\left(\frac{25}{9}\right)^{4x}}{\left(\frac{25}{9}\right)^{3-x}} = \frac{\left[\left(\frac{5}{3}\right)^4\right]^2}{\left[\left(\frac{3}{5}\right)^3\right]^2}$$

$$\frac{\left[\left(\frac{5}{3}\right)^2\right]^{4x}}{\left[\left(\frac{5}{3}\right)^2\right]^{3-x}} = \frac{\left(\frac{5}{3}\right)^8}{\left(\frac{3}{5}\right)^6}$$

$$\frac{\left(\frac{5}{3}\right)^{8x}}{\left(\frac{5}{3}\right)^{6-2x}} = \frac{\left(\frac{5}{3}\right)^8}{\left(\frac{5}{3}\right)^{-6}}$$

$$\left(\frac{5}{3}\right)^{8x-(6-2x)} = \left(\frac{5}{3}\right)^{8-(-6)}$$

$$\left(\frac{5}{3}\right)^{10x-6} = \left(\frac{5}{3}\right)^{14}$$

$$\left(\frac{a}{b}\right)^c = \left(\frac{b}{a}\right)^{-c}$$

$$\left(\frac{9}{25}\right)^{x-3} = \left(\frac{25}{9}\right)^{3-x}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$10x - 6 = 14$$

$$x = 2$$

b. $\frac{1}{4-2x} \geq \frac{8}{x}$

$$\frac{1}{4-2x} - \frac{8}{x} \geq 0$$

$$\frac{x - 8(4 - 2x)}{x(4 - 2x)} \geq 0$$

$$\frac{x - 32 + 16x}{x(4 - 2x)} \geq 0$$

$$\frac{17x - 32}{x(4 - 2x)} \geq 0$$

Critical value:

$$x = \frac{32}{17} \quad x = 0 \quad x = 2$$

x	$(-\infty, 0)$	$(0, \frac{32}{17})$	$(\frac{32}{17}, 2)$	$(2, \infty)$
$17x - 32$	-	-	+	+
$(4 - 2x)$	+	+	+	-
x	-	+	+	+
$\frac{17x - 32}{x(4 - 2x)}$	⊕	-	⊕	-

Solution: $\left\{x: x < 0 \cup \frac{32}{17} \leq x < 2\right\}$

3. a) The first three terms of a geometric series are $\left(3c - \frac{7}{2}\right)$, $(3c - 2)$ and 6.

Determine the value of c . Hence, find the seventh term of this series.

- b) Expand $\left(\frac{3}{2}x^2 - 1\right)^3$

SOLUTION

- a) Geometric series

$$\left(3c - \frac{7}{2}\right), (3c - 2) \text{ and } 6$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_2}$$

$$\frac{3c - 2}{3c - \frac{7}{2}} = \frac{6}{3c - 2}$$

$$(3c - 2)^2 = 6\left(3c - \frac{7}{2}\right)$$

$$9c^2 - 12c + 4 = 18c - 21$$

$$9c^2 - 30c + 25 = 0$$

$$(3c - 5)(3c - 5) = 0$$

$$c = \frac{5}{3}$$

$$a = 3c - \frac{7}{2}$$

$$= 3\left(\frac{5}{3}\right) - \frac{7}{2}$$

$$= \frac{3}{2}$$

$$r = \frac{6}{3c - 2}$$

$$= \frac{6}{3\left(\frac{5}{3}\right) - 2}$$

$$= 2$$

$$T_7 = ar^6$$

$$= \left(\frac{3}{2}\right)(2)^6$$

$$= 96$$

$$\begin{aligned} \text{b) } \left(\frac{3}{2}x^2 - 1\right)^3 &= \binom{3}{0}\left(\frac{3x^2}{2}\right)^3(-1)^0 + \binom{3}{1}\left(\frac{3x^2}{2}\right)^2(-1)^1 + \binom{3}{2}\left(\frac{3x^2}{2}\right)^1(-1)^2 + \binom{3}{3}\left(\frac{3x^2}{2}\right)^0(-1)^3 \\ &= (1)\left(\frac{27x^6}{8}\right)(1) + (3)\left(\frac{9x^4}{4}\right)(-1) + (3)\left(\frac{3x^2}{2}\right)(1) + (1)(1)(-1) \\ &= \frac{27}{8}x^6 - \frac{27}{4}x^4 + \frac{9}{2}x^2 - 1 \end{aligned}$$

i. a) Given the matrix $\begin{bmatrix} 1 & 3 & 4 \\ a+2b & 3 & 2 \\ 4 & a+b & 9 \end{bmatrix}$ such that $M_{11} = 7$ and $C_{12} = -1$,

calculate the values of a and b.

b) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 13 & -6 \\ -1 & -7 & 2 \\ -2 & 2 & 0 \end{bmatrix}$ and $C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

i. Find determinant of A by expanding first column.

ii. Evaluate $(A^2 - B^T)C$.

SOLUTION

a) $\begin{bmatrix} 1 & 3 & 4 \\ a+2b & 3 & 2 \\ 4 & a+b & 9 \end{bmatrix}$

$$M_{11} = \begin{vmatrix} 3 & 2 \\ a+b & 9 \end{vmatrix}$$

$$= 27 - 2a - 2b$$

$$27 - 2a - 2b = 7$$

$$2a + 2b = 20$$

$$a + b = 10 \quad \dots\dots\dots (1)$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} a+2b & 2 \\ 4 & 9 \end{vmatrix}$$

$$= (-1)[9a + 18b - 8]$$

$$= -9a - 18b + 8$$

$$-9a - 18b + 8 = -1$$

$$9a + 18b = 9$$

$$a + 2b = 1 \quad \dots\dots\dots (2)$$

$$(2) - (1)$$

$$b = 1 - 10 = -9$$

$$a + (-9) = 10$$

$$a = 19$$

$$\therefore a = 19, b = -9$$

bi)

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{bmatrix}, B = \begin{bmatrix} 7 & 13 & -6 \\ -1 & -7 & 2 \\ -2 & 2 & 0 \end{bmatrix} \text{ and } C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{aligned} |A| &= (1) \begin{vmatrix} 3 & 2 \\ 10 & 9 \end{vmatrix} - (1) \begin{vmatrix} 3 & 4 \\ 10 & 9 \end{vmatrix} + (4) \begin{vmatrix} 3 & 4 \\ 3 & 2 \end{vmatrix} \\ &= (1)[27 - 20] - (1)[27 - 40] + (4)[6 - 12] \\ &= 7 + 13 - 24 \\ &= -4 \end{aligned}$$

$$\begin{aligned}\text{bii) } (A^2 - B^T)C &= \left[\begin{pmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 \\ 1 & 3 & 2 \\ 4 & 10 & 9 \end{pmatrix} - \begin{pmatrix} 7 & -1 & -2 \\ 13 & -7 & 2 \\ -6 & 2 & 0 \end{pmatrix} \right] \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \\ &= \left[\begin{pmatrix} 20 & 52 & 46 \\ 12 & 32 & 28 \\ 50 & 132 & 117 \end{pmatrix} - \begin{pmatrix} 7 & -1 & -2 \\ 13 & -7 & 2 \\ -6 & 2 & 0 \end{pmatrix} \right] \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \\ &= \left[\begin{pmatrix} 13 & 53 & 48 \\ -1 & 39 & 26 \\ 56 & 130 & 117 \end{pmatrix} \right] \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 162 \\ 90 \\ 420 \end{pmatrix}\end{aligned}$$

5. a) Given $f(x) = \left(\frac{5x+1}{4x}\right)$ and $g(x) = \frac{\sqrt{x+1}-2}{x^2-4}$. Find

i. The domain of $g(x)$.

ii. $h(x)$, if $(f \circ h)(x) = x$

b) Given $p(x) = \ln(3x + 6)$ and $q(x) = \frac{e^x}{3} - 2$. Show that $p(x)$ and $q(x)$ are

inverses of each other.

SOLUTION

$$f(x) = \left(\frac{5x + 1}{4x}\right)$$

$$g(x) = \frac{\sqrt{x+1}-2}{x^2-4}$$

5ai) Domain of $g(x)$

$$x + 1 \geq 0 \quad \text{and} \quad x^2 - 4 \neq 0$$

$$x \geq -1 \quad \text{and} \quad x \neq \pm 2$$

$$\therefore D_f: [-1, 2) \cup (2, \infty)$$

5a ii) $(f \circ h)(x) = x$

$$f[h(x)] = x$$

$$\left[\frac{5h(x) + 1}{4h(x)}\right] = x$$

$$5h(x) + 1 = 4xh(x)$$

$$5h(x) - 4xh(x) = -1$$

$$h(x)[5 - 4x] = -1$$

$$h(x) = \frac{-1}{5 - 4x}$$

5b) Given $p(x) = \ln(3x + 6)$ and $q(x) = \frac{e^x}{3} - 2$

$$\begin{aligned} p[q(x)] &= \ln\left[3\left(\frac{e^x}{3} - 2\right) + 6\right] \\ &= \ln[e^x - 6 + 6] \\ &= \ln[e^x] \\ &= x \ln[e] \\ &= x \end{aligned}$$

$$\begin{aligned} q[p(x)] &= \frac{e^{\ln(3x+6)}}{3} - 2 \\ &= \frac{3x + 6}{3} - 2 \\ &= \frac{3x + 6 - 6}{3} \\ &= x \end{aligned}$$

Since $p[q(x)] = q[p(x)] = x$, therefore $p(x)$ and $q(x)$ are inverses of each other

6. The polynomial $P(x) = x^4 + ax^3 - 7x^2 - 4ax + b$ has a factor $(x + 3)$ and remainder 60 when divided by $(x - 3)$. Find the values of a and b . Hence, factorise $P(x)$ completely.

SOLUTION

$$P(x) = x^4 + ax^3 - 7x^2 - 4ax + b$$

$$P(-3) = 0$$

$$P(3) = 60$$

$$P(-3) = (-3)^4 + a(-3)^3 - 7(-3)^2 - 4a(-3) + b = 0$$

$$81 - 27a - 63 + 12a + b = 0$$

$$15a - b = 18 \quad \dots\dots\dots (1)$$

$$P(3) = (3)^4 + a(3)^3 - 7(3)^2 - 4a(3) + b = 60$$

$$81 + 27a - 63 - 12a + b = 60$$

$$15a + b = 42 \quad \dots\dots\dots (2)$$

$$(2) - (1)$$

$$2b = 24$$

$$b = 12$$

$$15a - 12 = 18$$

$$a = 2$$

$$\therefore a = 2, b = 12$$

$$P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$$

$$\begin{array}{r}
 x^3 - x^2 - 4x + 4 \\
 x + 3 \overline{) x^4 + 2x^3 - 7x^2 - 8x + 12} \\
 \underline{x^4 + 3x^3} \\
 -x^3 - 7x^2 - 8x + 12 \\
 \underline{-x^3 - 3x^2} \\
 -4x^2 - 8x + 12 \\
 \underline{-4x^2 - 12x} \\
 4x + 12 \\
 \underline{4x + 12} \\
 0
 \end{array}$$

$$\begin{aligned}
 P(x) &= (x + 3)(x^3 - x^2 - 4x + 4) \\
 &= (x + 3)[x^2(x - 1) - 4(x - 1)] \\
 &= (x + 3)(x - 1)[x^2 - 4] \\
 &= (x + 3)(x - 1)(x + 2)(x - 2)
 \end{aligned}$$

7. a) Express $12 \cos \theta + 7 \sin \theta$ in the form of $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$
- b) Hence, show that the maximum value of $\frac{1}{12 \cos \theta + 7 \sin \theta + 15}$ is $\frac{1}{32}(15 + \sqrt{193})$.

SOLUTION

$$7a) \quad 12 \cos \theta + 7 \sin \theta = R \cos(\theta - \alpha)$$

$$12 \cos \theta + 7 \sin \theta = R[\cos \theta \cos \alpha + \sin \theta \sin \alpha]$$

$$12 \cos \theta + 7 \sin \theta = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$R \cos \alpha = 12 \quad \dots\dots\dots (1)$$

$$R \sin \alpha = 7 \quad \dots\dots\dots (2)$$

$$(1)^2 + (2)^2$$

$$R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 12^2 + 7^2$$

$$R^2(\cos^2 \alpha + \sin^2 \alpha) = 193$$

$$R^2(1) = 193$$

$$R = \sqrt{193}$$

$$(2) \div (1)$$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{7}{12}$$

$$\tan \alpha = \frac{7}{12}$$

$$\alpha = 30.3^\circ$$

$$12 \cos \theta + 7 \sin \theta = \sqrt{193} \cos(\theta - 30.3^\circ)$$

$$7b) \quad \frac{1}{12 \cos \theta + 7 \sin \theta + 15} = \frac{1}{(\sqrt{193} \cos(\theta - 30.3^\circ)) + 15}$$

$$-1 \leq \cos(\theta - 30.3^\circ) \leq 1$$

$$-\sqrt{193} \leq \sqrt{193} \cos(\theta - 30.3^\circ) \leq \sqrt{193}$$

$$-\sqrt{193} + 15 \leq \sqrt{193} \cos(\theta - 30.3^\circ) + 15 \leq \sqrt{193} + 15$$

$$\frac{1}{\sqrt{193} + 15} \leq \frac{1}{\sqrt{193} \cos(\theta - 30.3^\circ) + 15} \leq \frac{1}{-\sqrt{193} + 15}$$

$$\frac{1}{\sqrt{193} + 15} \leq \frac{1}{12 \cos \theta + 7 \sin \theta + 15} \leq \frac{1}{-\sqrt{193} + 15}$$

The Maximum value of $\frac{1}{12 \cos \theta + 7 \sin \theta + 15}$

$$\begin{aligned} \frac{1}{-\sqrt{193} + 15} &= \frac{1}{(15 - \sqrt{193})} \cdot \frac{(15 + \sqrt{193})}{(15 + \sqrt{193})} \\ &= \frac{15 + \sqrt{193}}{225 + 15\sqrt{193} - 15\sqrt{193} - 193} \\ &= \frac{15 + \sqrt{193}}{32} \\ &= \frac{1}{32} (15 + \sqrt{193}) \end{aligned}$$

8. The function $g(x)$ is defined by

$$g(x) = \begin{cases} 2 & , x \leq 2 \\ \frac{x-2}{\sqrt{2x}-2} & , 2 < x \leq 8 \\ \frac{|8-x|}{x-8} & , x > 8 \end{cases}$$

Find

a) $\lim_{x \rightarrow 2^+} g(x)$

b) $\lim_{x \rightarrow 8^+} g(x)$

SOLUTION

$$\begin{aligned} \text{a) } \lim_{x \rightarrow 2^+} g(x) &= \lim_{x \rightarrow 2^+} \left(\frac{x-2}{\sqrt{2x}-2} \right) \\ &= \lim_{x \rightarrow 2^+} \left(\frac{x-2}{\sqrt{2x}-2} \right) \left(\frac{\sqrt{2x}+2}{\sqrt{2x}+2} \right) \\ &= \lim_{x \rightarrow 2^+} \left(\frac{(x-2)(\sqrt{2x}+2)}{2x+2\sqrt{2x}-2\sqrt{2x}-4} \right) \\ &= \lim_{x \rightarrow 2^+} \left(\frac{(x-2)(\sqrt{2x}+2)}{2x-4} \right) \\ &= \lim_{x \rightarrow 2^+} \left(\frac{(x-2)(\sqrt{2x}+2)}{2(x-2)} \right) \\ &= \lim_{x \rightarrow 2^+} \left[\frac{(\sqrt{2x}+2)}{2} \right] \\ &= \frac{\sqrt{2(2)}+2}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{b) } |8 - x| &= \begin{cases} 8 - x & 8 - x \geq 0 \\ -(8 - x) & 8 - x < 0 \end{cases} \\ &= \begin{cases} 8 - x & x \leq 8 \\ -(8 - x) & x > 8 \end{cases} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 8^+} g(x) &= \lim_{x \rightarrow 8^+} \frac{|8 - x|}{x - 8} \\ &= \lim_{x \rightarrow 8^+} \frac{-(8 - x)}{x - 8} \\ &= \lim_{x \rightarrow 8^+} \frac{x - 8}{x - 8} \\ &= \lim_{x \rightarrow 8^+} 1 \\ &= 1 \end{aligned}$$

9. a) Find the derivative of $f(x) = \frac{6}{\sqrt{x}}$ using the first principle.
- b) Find the value of $\frac{dy}{dx}$ when $x = 0$ for each of the following:
- $y = \ln(9 - 2x)$
 - $y = \frac{e^{-3x}}{\sqrt{3x+1}}$

SOLUTION

a) $f(x) = \frac{6}{\sqrt{x}}$

$$f(x+h) = \frac{6}{\sqrt{x+h}}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{6}{\sqrt{x+h}} - \frac{6}{\sqrt{x}}}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{6}{\sqrt{x+h}} - \frac{6}{\sqrt{x}} \right) \cdot \left(\frac{1}{h} \right)$$

$$= \lim_{h \rightarrow 0} \left(\frac{6\sqrt{x} - 6\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} \right) \cdot \left(\frac{1}{h} \right)$$

$$= \lim_{h \rightarrow 0} \left[\frac{6(\sqrt{x} - \sqrt{x+h})}{\sqrt{x}\sqrt{x+h}} \right] \cdot \left(\frac{1}{h} \right)$$

$$= \lim_{h \rightarrow 0} \left[\frac{6(\sqrt{x} - \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \right] \cdot \left(\frac{1}{h} \right)$$

$$= \lim_{h \rightarrow 0} \left[\frac{6(x + \sqrt{x}\sqrt{x+h} - \sqrt{x}\sqrt{x+h} - (x+h))}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \right] \cdot \left(\frac{1}{h} \right)$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \left[\frac{6(-h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \right] \cdot \left(\frac{1}{h} \right) \\
 &= \lim_{h \rightarrow 0} \left[\frac{-6}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} \right] \\
 &= \frac{-6}{\sqrt{x}\sqrt{x+0}(\sqrt{x} + \sqrt{x+0})} \\
 &= \frac{-6}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})} \\
 &= \frac{-6}{x(2\sqrt{x})} \\
 &= \frac{-3}{x^2}
 \end{aligned}$$

bi) $y = \ln(9 - 2x)$

$$\frac{dy}{dx} = \frac{1}{9 - 2x} \frac{d}{dx}(9 - 2x)$$

$$\frac{dy}{dx} = \frac{1}{9 - 2x}(-2)$$

$$\frac{dy}{dx} = \frac{-2}{9 - 2x}$$

When $x = 0$

$$\frac{dy}{dx} = \frac{-2}{9 - 0}$$

$$= -\frac{2}{9}$$

bii) $y = \frac{e^{-3x}}{\sqrt{3x+1}}$

$$u = e^{-3x}$$

$$u' = -3e^{-3x}$$

$$v = \sqrt{3x+1} = (3x+1)^{\frac{1}{2}}$$

$$v' = \frac{1}{2}(3x+1)^{-\frac{1}{2}} \frac{d}{dx}(3x+1)$$

$$= \frac{3}{2(3x+1)^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(3x+1)^{\frac{1}{2}}(-3e^{-3x}) - e^{-3x} \left(\frac{3}{2(3x+1)^{\frac{1}{2}}} \right)}{\left[(3x+1)^{\frac{1}{2}} \right]^2}$$

When $x = 0$;

$$\frac{dy}{dx} = \frac{(0+1)^{\frac{1}{2}}(-3e^0) - e^0 \left(\frac{3}{2(0+1)^{\frac{1}{2}}} \right)}{\left[(0+1)^{\frac{1}{2}} \right]^2}$$

$$= \frac{-3 - \frac{3}{2}}{1}$$

$$= -\frac{9}{2}$$

10. Given $f(x) = \frac{3x}{x^2+9}$, where $x > 0$. Find the coordinates of the stationary point and state it's nature.

SOLUTION

$$f(x) = \frac{3x}{x^2 + 9}$$

$$u = 3x \qquad v = x^2 + 9$$

$$u' = 3 \qquad v' = 2x$$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2 + 9)(3) - (3x)(2x)}{(x^2 + 9)^2}$$

$$= \frac{3x^2 + 27 - 6x^2}{(x^2 + 9)^2}$$

$$= \frac{-3x^2 + 27}{(x^2 + 9)^2}$$

$$u = -3x^2 + 27 \qquad v = (x^2 + 9)^2$$

$$u' = -9x \qquad v' = 2(x^2 + 9) \frac{d}{dx}(x^2 + 9)$$

$$= 2(x^2 + 9)(2x)$$

$$= 4x(x^2 + 9)$$

$$f''(x) = \frac{vu' - uv'}{v^2}$$

$$\begin{aligned} &= \frac{(x^2 + 9)^2(-9x) - (-3x^2 + 27)4x(x^2 + 9)}{(x^2 + 9)^4} \\ &= \frac{-9x(x^2 + 9)^2 - 4x(-3x^2 + 27)(x^2 + 9)}{(x^2 + 9)^4} \end{aligned}$$

Let $f'(x) = 0$

$$\frac{-3x^2 + 27}{(x^2 + 9)^2} = 0$$

$$-3x^2 + 27 = 0$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$x = \pm 3$$

Since $x > 0$, $x = 3$

When $x = 3$

$$f(x) = \frac{3x}{x^2 + 9}$$

$$= \frac{9}{9 + 9}$$

$$= \frac{1}{2}$$

$\left(3, \frac{1}{2}\right)$ is a stationary point

$$f''(x) = \frac{-9x(x^2 + 9)^2 - 4x(-3x^2 + 27)(x^2 + 9)}{(x^2 + 9)^4}$$

$$= \frac{-27(9 + 9)^2 - 12(-27 + 27)(9 + 9)}{(9 + 9)^4}$$

$$= \frac{-27(9 + 9)^2 - 0}{(9 + 9)^4}$$

$$< 0 \text{ (Max)}$$

$\therefore \left(3, \frac{1}{2}\right)$ is a maximum point